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$\bar{Z}$-dependence of partial and total photon interactions in some biological samples

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Abstract

The effective atomic numbers for total and partial photon interactions of 11 different biological samples have been determined using a new approximation in the energy region 10–150 keV. In this study, 'mass attenuation coefficients' for the various individual interactions have been calculated. The effective atomic numbers have been obtained by considering the $\bar{Z}$-dependence of photon cross-sections and the variation of $\bar{Z}$-exponents with energy. © 2000 Elsevier Science Ltd. All rights reserved.

1. Introduction

The effective atomic number ($\bar{Z}$) is a useful parameter for the interpretation of the attenuation of X-ray or gamma radiation by a complex medium like a biological tissue. Especially, studies on the interaction of low energy photons with biological samples are of importance in the field of diagnostic radiology and nuclear medicine, build-up and dose calculations.

At low photon energies (<1.02 MeV), three partial processes (namely, coherent scattering, incoherent scattering and photoelectric effect) contribute to the total interaction (Hubbell, 1999a, 1999b). Hine (1952) pointed out that the effective atomic numbers in materials composed of several elements cannot be represented uniquely and the contribution due to each interaction process has to be taken into account differently.

Conventionally, the elemental cross-section per atom is defined to be directly proportional to Z^m. The Z-exponent m depends on both the type of process considered and the photon energy. Therefore, the atomic cross-section of an element for a partial process is generally expressed as:

$$\sigma_a = K(E)Z^m \tag{1}$$

The m values for a tissue have varied in the range of 4–5 for the photoelectric effect; 2–3 for the coherent Rayleigh scattering; ≈ 1 for the incoherent Compton scattering and 1–2 for the pair production. $K(E)$ is a function of photon energy for each partial process. In the energy region from 10 to 150 keV, pair production does not exist, but the other three partial processes contribute significantly to the total interaction process. Hence, it is appropriate to define an effective atomic number for the total photon interaction corresponding to these three partial processes.

The mass attenuation coefficient for the individual element is given by:

$$\frac{\mu}{\rho} = \sigma_a\left(\frac{N_A}{A}\right) = K(E)Z^m\left(\frac{N_A}{A}\right)$$

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Table 1
The values of $\bar{Z}$ exponent m and $K(E)$ for partial photon interaction processes

E (MeV)	m_1	K_{Pho}	m_2	K_{coh}	m_3	K_{incoh}
0.010	4.33	1.726E−02	2.47	4.202E−02	0.757	0.688
0.015	4.44	3.810E−03	2.46	2.524E−02	0.824	0.687
0.020	4.51	1.311E−03	2.46	1.627E−02	0.857	0.683
0.030	4.60	2.942E−04	2.51	7.765E−03	0.896	0.660
0.040	4.65	1.027E−04	2.55	4.375E−03	0.922	0.630
0.050	4.69	4.559E−05	2.58	2.784E−03	0.940	0.602
0.060	4.72	2.363E−05	2.60	1.922E−03	0.953	0.577
0.080	4.76	8.404E−06	2.62	1.107E−03	0.968	0.538
0.100	4.79	3.808E−06	2.64	6.732E−04	0.977	0.507
0.150	4.83	9.243E−07	2.67	2.910E−04	0.988	0.451

$$= K(E) n_a Z^{m-1} \tag{2}$$

where N_A, Z and A are the Avogadro constant, atomic number and atomic weight, respectively; $n_a = Z(N_A/A)$ is the number of electrons per unit mass of the element of atomic number Z.

If the absorber consists of a chemical compound or a homogeneous mixture, the mass attenuation coefficient can be obtained using the mixture rule. Thus, if $\bar{Z}$ represents the effective atomic number of the compound in analogy with Eq. (2), the mass attenuation coefficient for a compound can be written as:

$$\left(\frac{\mu}{\rho}\right)_{\text{comp.}} = K(E) n_0 \bar{Z}^{m-1} \tag{3}$$

where n_0 is the number of electrons per unit mass of the compound. With m_1 and m_2 as exponents of the effective atomic numbers for photoelectric effect and coherent scattering, respectively, the total mass attenuation coefficient for a compound is given by:

$$\left(\frac{\mu}{\rho}\right)_{\text{comp.}} = n_0\left[K_1(E)\bar{Z}_1^{m_1-1} + K_2(E)\bar{Z}_2^{m_2-1} + \sigma_e^{\text{KN}}\right] \tag{4}$$

where σ_e^{KN} is the Klein–Nishina cross-section.

Effective atomic numbers for photoelectric effect ($\bar{Z}_1$) and coherent scattering ($\bar{Z}_2$) are given by:

$$\bar{Z}_1 = \left[\sum_i \lambda_i Z_i^{m_1-1}\right]^{1/(m_1-1)} \tag{5a}$$

and

$$\bar{Z}_2 = \left[\sum_i \lambda_i Z_i^{m_2-1}\right]^{1/(m_2-1)} \tag{5b}$$

where λ_i is the relative electron content of the ith element in the compound, in other words the fractional electron number. If the contribution is small because of coherent scattering, the compound can be characterized with a single effective number $\bar{Z}$. But the incoherent cross-section term in Eq. (4) (i.e., the third term) is strictly valid only for a free electron behavior or for gamma energies much larger than the electron binding energies.

Analysis of $\sigma_e^{\text{KN}}(E)$ and $\sigma_a^{\text{incoh}}/Z$ has been made by Jackson and Hawkes (1981). They have pointed out that there are substantial discrepancies between the variation of $\sigma_e^{\text{KN}}(E)$ and $\sigma_a^{\text{incoh}}/Z$ for $E < 150$ keV and for light elements. Moreover, for heavy elements the discrepancies persist up to much higher energies. Even at 400 keV it has been determined that the discrepancy is still $\approx 2\%$ for $Z = 53$ and $> 1\%$ for $Z = 20$ and that the discrepancies are much greater at lower energies. Thus, it can be said that the incoherent cross-section term in Eq. (4) is not valid in the energy region of 10–150 keV.

Table 2
Partial effective atomic numbers for biological samples

Energy (MeV)	Bone			Muscle			Water		
	Photo	Coherent	Incoherent	Photo	Coherent	Incoherent	Photo	Coherent	Incoherent
0.010	13.79	11.19	7.95	7.67	6.62	5.14	7.60	6.67	5.09
0.015	13.98	11.63	8.77	7.70	6.61	5.57	7.60	6.64	5.54
0.020	14.09	11.78	9.12	7.71	6.69	5.85	7.60	6.72	5.86
0.030	14.23	11.81	9.43	7.72	6.77	6.08	7.60	6.80	6.11
0.040	14.33	11.85	9.66	7.73	6.79	6.13	7.60	6.82	6.16
0.050	14.39	11.93	9.94	7.74	6.81	6.17	7.60	6.84	6.20
0.060	14.44	12.00	10.10	7.74	6.83	6.15	7.60	6.86	6.17
0.080	14.51	12.08	10.43	7.75	6.86	6.25	7.60	6.89	6.28
0.100	14.56	12.12	10.61	7.76	6.87	6.30	7.59	6.90	6.33
0.150	14.64	12.15	10.90	7.77	6.89	6.31	7.59	6.91	6.33

2. Methods

In view of the above observations, the total mass attenuation coefficient for a compound can be expressed as:

$$\left(\frac{\mu}{\rho}\right)_{\text{comp.}} = n_0\Big[K_1(E)\bar{Z}_1^{m_1-1} + K_2(E)\bar{Z}_2^{m_2-1} + K_3(E)\bar{Z}_3^{m_3-1}\Big] \quad (6)$$

In the present work, taking into account the binding effect, the $\bar{Z}$-exponents m and $\bar{Z}$ have been determined for 11 biological samples for all photon interaction processes (i.e., photoelectric effect, coherent and incoherent scattering) in the energy region of 10–150 keV.

To deduce the effective atomic number using Eq. (6), firstly $\bar{Z}$-exponents m and $K(E)$ for various biological materials should be obtained. To determine m and $K(E)$ values from Eq. (1) for the elements, a least-squares analysis was made. In this analysis, a linear relation of $\ln(\sigma_a)$ vs. $\ln(Z)$ was assumed for different energies. Elements having Z from 4 to 16 were used for the analysis. The partial cross-sections for each individual element were taken from XCOM (Berger and Hubbell, 1987/1999). μ/ρ values of biological samples for each interaction were computed from the elemental data, using fractions by weight given by Kim (1974a, 1974b).

Using the calculated values of $(\mu/\rho)_{\text{comp.}}$, $K(E)$, n_0 and m, effective atomic numbers were determined for partial photon interaction processes in biological samples for photon energies between 10 and 150 keV.

Yang et al. (1987) has derived formulas between effective atomic numbers for partial and total inter-

Table 3
The comparison of the calculated total mass attenuation coefficients for muscle and water (m^2/kg)

E (MeV)	Water		Muscle	
	$(\mu/\rho)^a$	$(\mu/\rho)^b$	$(\mu/\rho)^a$	$(\mu/\rho)^b$
0.010	0.5330	0.5330	0.5447	0.5446
0.015	0.1673	0.1673	0.1717	0.1717
0.020	0.0810	0.0810	0.0830	0.0830
0.030	0.0376	0.0376	0.0381	0.0381
0.040	0.0268	0.0268	0.0270	0.0270
0.050	0.0227	0.0227	0.0227	0.0227
0.060	0.0206	0.0206	0.0205	0.0205
0.080	0.0184	0.0184	0.0183	0.0183
0.100	0.0171	0.0171	0.0170	0.0170
0.150	0.0151	0.0151	0.0149	0.0149

[a] By means of Eq. (6).
[b] By means of the mixture rule.

Table 4
Total effective atomic numbers of various human tissues

Energy (MeV)	Bone $\bar{Z}$	Brain $\bar{Z}$	Spleen $\bar{Z}$	Kidney $\bar{Z}$	Muscle $\bar{Z}$	Liver $\bar{Z}$	Lung $\bar{Z}$	Tongue $\bar{Z}$	Heart $\bar{Z}$	Pancreas $\bar{Z}$	Ovary $\bar{Z}$	Water $\bar{Z}$
0.010	13.77	7.52	7.46	7.64	7.65	7.79	7.59	7.38	7.42	7.51	7.51	7.58
0.015	13.95	7.52	7.46	7.64	7.66	7.80	7.59	7.38	7.42	7.50	7.50	7.57
0.020	14.05	7.52	7.45	7.64	7.66	7.80	7.58	7.37	7.41	7.49	7.49	7.56
0.030	14.15	7.51	7.43	7.63	7.65	7.79	7.57	7.36	7.39	7.47	7.47	7.54
0.040	14.21	7.50	7.42	7.62	7.64	7.78	7.56	7.35	7.38	7.46	7.46	7.53
0.050	14.23	7.49	7.41	7.61	7.63	7.77	7.55	7.33	7.37	7.45	7.45	7.52
0.060	14.24	7.49	7.41	7.61	7.63	7.77	7.54	7.33	7.37	7.45	7.45	7.52
0.080	14.24	7.47	7.39	7.59	7.61	7.57	7.53	7.31	7.35	7.44	7.43	7.51
0.100	14.20	7.44	7.37	7.57	7.59	7.72	7.51	7.29	7.33	7.42	7.42	7.49
0.150	14.07	7.41	7.36	7.55	7.56	7.69	7.48	7.26	7.31	7.41	7.41	7.25
Average	14.11	7.49	7.42	7.61	7.63	7.75	7.55	7.34	7.38	7.46	7.46	7.51
$\pm\sigma$	0.15	0.04	0.04	0.03	0.03	0.07	0.04	0.04	0.04	0.03	0.03	0.09

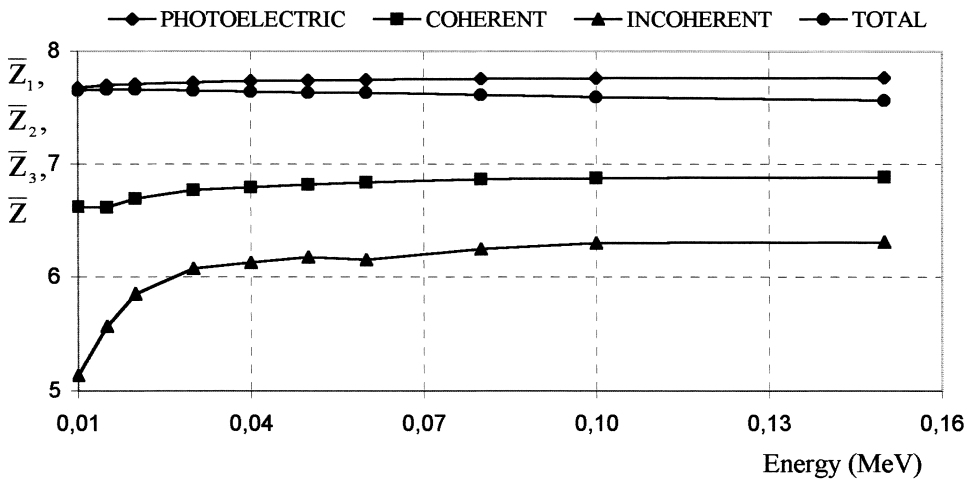


Fig. 1. Variation of effective atomic numbers for partial and total photon interactions in muscle.

actions. Similarly, we can write a formula for the partial and total effective atomic numbers based on the mass attenuation coefficients, as follows:

$$K_1(E)\bar{Z}^{m_1} + K_2(E)\bar{Z}^{m_2} + K_3(E)\bar{Z}^{m_3} = K_1(E)\bar{Z}_1^{m_1} + K_2(E)\bar{Z}_2^{m_2} + K_3(E)\bar{Z}_3^{m_3} \quad (7)$$

where $\bar{Z}_1$, $\bar{Z}_2$, $\bar{Z}_3$ and $\bar{Z}$ are effective atomic numbers for photoelectric, coherent, incoherent and total interactions, respectively. In this way, total effective atomic numbers have been determined.

3. Conclusions and discussion

Using the elemental data in the XCOM (Berger and Hubbell, 1987/1999), the values of $\bar{Z}$-exponent m and $K(E)$ have been calculated by means of the least-squares method and the results are given in Table 1.

For most biological samples the effective atomic numbers for all processes lie between 5 and 15. Therefore, the $Z = 4$–16 group of elements has been considered for analysis. From Table 1, it is clear that the exponent value increases with the energy for all the partial processes; namely, the m_1 parameter for photoelectric interaction varies from 4.33 to 4.83, the m_2 parameter for coherent scattering from 2.47 to 2.67 and the m_3 parameter for incoherent scattering from 0.757 to 0.988. The exponents not only vary with energy but also become different at a given energy as the elemental composition changes. However, the same exponent value was used in Eq. (6) for all biological samples at the same energy. It is possible to character-

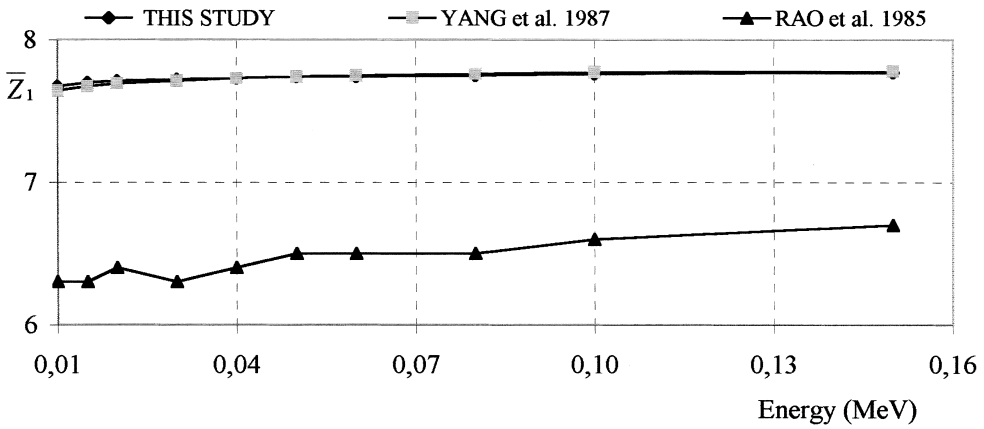


Fig. 2. Variation of effective atomic number for photoelectric interaction in muscle.

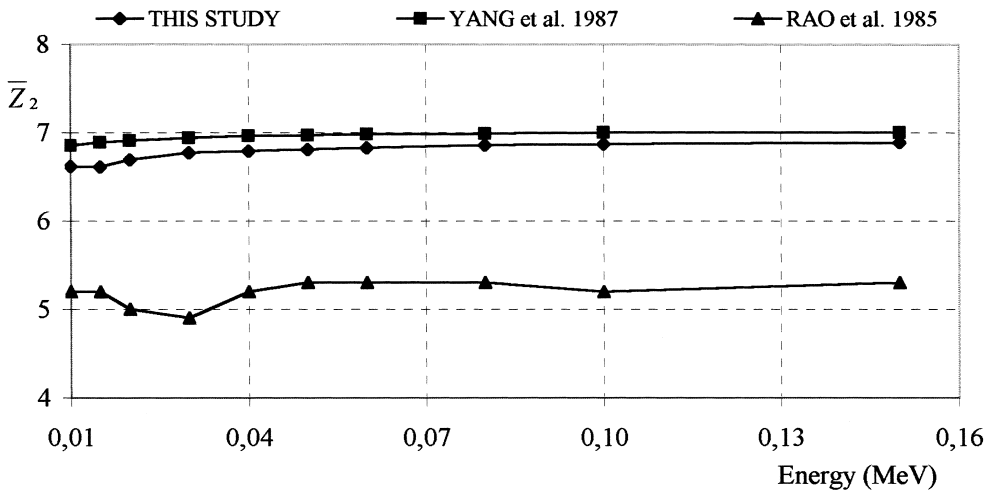


Fig. 3. Variation of effective atomic number for coherent scattering in muscle.

ize m and $K(E)$ for different photon energies by choosing a suitable power series and fitting a suitable number of parameters. Using evaluated m_i and $K_i(E)$ values ($i = 1$, 2 and 3), depending on E (MeV) and μ/ρ (barn/atom), the following equations are obtained:

$$m_1 = 0.731 - 1.879E + 4.983E^{0.069}$$

$$m_2 = 0.595 - 0.948E + 2.578E^{0.080}$$

(in the energy region 0.020–0.150 MeV)

$$m_3 = 1.083 - 0.118E - 2.793 \times 10^{-2}E^{-0.532}$$

$$K_1 = 8.7 \times 10^{-10}E^{-3.638}$$

$$K_2 = 2.244 \times 10^{-3}E^{-0.743}\exp(-54.854E) + 3.252 \times 10^{-3}\exp(-16.197E)$$

$$K_3 = 4.997 \times 10^2 - \frac{4.993 \times 10^2}{E^{7.545\times10^{-4}}} + \frac{2.867}{\exp(3.320E^{0.494})}$$

From Eq. (3) and the values given in Table 1, partial $\bar{Z}$ values were calculated for bone, muscle and water as given in Table 2. Moreover, calculated total mass attenuation coefficients for muscle and water by means

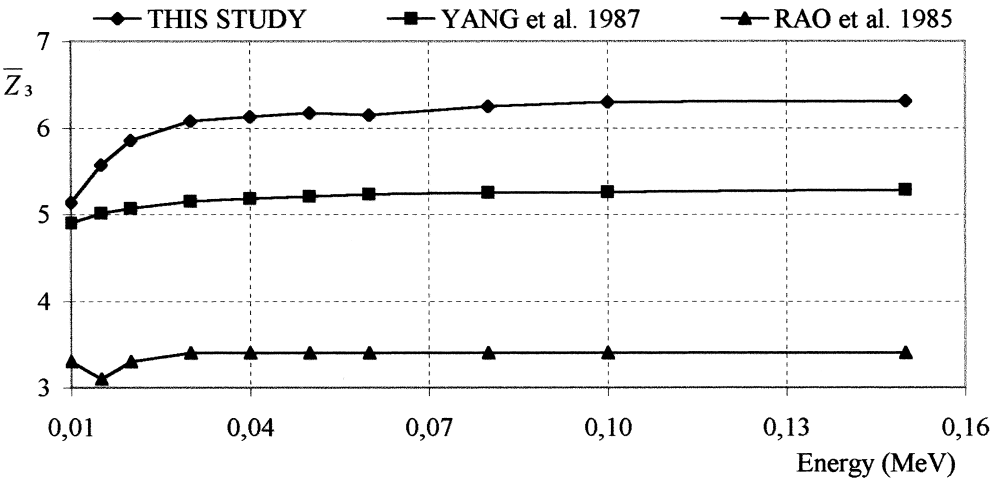


Fig. 4. Variation of effective atomic number for incoherent scattering in muscle.

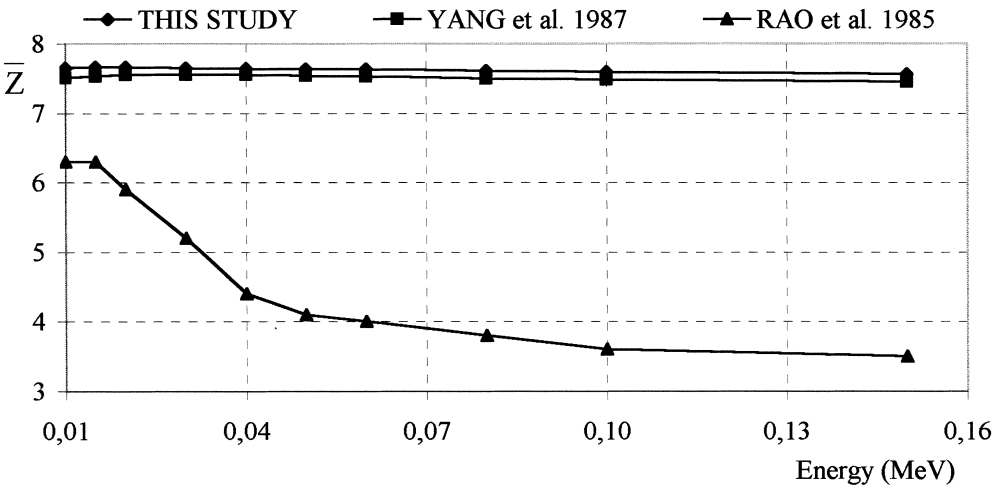


Fig. 5. Variation of effective atomic number for total photon interaction in muscle.

of Eq. (6) and the mixture rule are compared in Table 3. It can be seen that the results are in good agreement.

Effective atomic numbers of various human tissues for total mass attenuation coefficients have been obtained by application of the Newton–Raphson method and Eq. (7), and the results are given in Table 4. The results of effective atomic numbers for partial and total photon interaction processes for muscle have been plotted in Fig. 1 for different photon energies. From Fig. 1 it is clear that the effective atomic number for photoelectric interaction is nearly constant in the energy region of 10–150 keV. In the case of coherent scattering, the effective atomic number decreases slightly by about 15 keV and then seems to be nearly constant between 30 and 150 keV photon energies. In the case of incoherent scattering, the effective atomic number increases with energy increasing from 10 to 30 keV and then seems to be nearly constant between 30 and 150 keV photon energies. In the case of total interaction, the total effective atomic number stays approximately constant and the variations with energy are within 3%.

In addition, the differences to be seen in the $\dot{Z}$ of biological samples depend on the constituent elements varying from H ($Z = 1$) to Ca ($Z = 20$) in biological samples. Bone has a different behavior from other biological samples because of the fact that its Ca and P contents are very high.

The behavior of $\dot{Z}$ with respect to energy for partial and total photon interactions in muscle have been plotted in Figs. 2–5. The variations in $\dot{Z}$ values found by Rao et al. (1985), using cross-sections per atom, and Yang et al. (1987), using cross-sections per electron, have also been given in these figures. Rao et al. (1985) reported that the effective atomic numbers for all partial interactions increase slightly with energy, whereas the total effective atomic number decreases by about 50%. However, the effective atomic number for the photoelectric interaction in this energy region has always a maximum; for incoherent scattering it has always a minimum. Also, Yang et al. (1987) reported that it is more appropriate to define an effective atomic number for the total interaction process, and that it is also more appropriate to use the total photon interaction cross-section per electron instead of the cross-section per atom of tissue. In this study, mass attenuation coefficients (m^2/kg) are used so that it is more realistic to determine effective atomic numbers.

Table 4 indicates that one can determine a $\dot{Z}$ which is not dependent on the photon energy, for the tissue samples considered here, in the energy region of interest. Therefore, the mean values of $\dot{Z}$, given in this table, may be used to represent these samples.

Consequently, the effective atomic number of a complex medium depends on the type of interaction. Also, the value for each type of interaction gives different results depending on the use of cross-sections (per electron, atom or unit mass). The actual value of $\dot{Z}$ is very sensitive to the weight fractions of the elements and the interpolation method used.

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