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RESEARCH ARTICLE

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Performance of scientific law-inspired optimization algorithms for constrained engineering applications

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ABSTRACT

The present work compares the performance of scientific law-inspired optimization algorithms for real-life constrained optimization applications. Ten such scientific law-inspired algorithms developed during the past decade are considered in this article. A constrained engineering application of the Stirling heat engine system is investigated with these algorithms. Four operating variables and two output constraints of the Stirling heat engine are considered for optimization. Comparative results are presented with statistical data to judge the performance of the algorithms and subsequently to identify the statistical significance and rank of each algorithm. The effects of various constraint handling methods on the performance of the algorithms are evaluated and presented. The behaviour of the constraint handling methods is analysed and presented. The effect of output constraints on the performance of the algorithms is also evaluated and presented. Finally, the convergence behaviour of the competitive algorithms is obtained and demonstrated.

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Optimization; metaheuristic algorithms; scientific law-based algorithms; statistical analysis; constraint handling techniques

1. Introduction

Optimization algorithms can be classified as either individual search algorithms or population-based algorithms. Population-based algorithms have gained in popularity owing to their adaptability to any kind of problem and application. The population-based algorithms are inspired by the various natural phenomena and laws or the intelligent behaviour of species in nature. Based on such inspiration, the population-based algorithms are categorized as swarm intelligence-based algorithms, evolutionary algorithms and scientific law-inspired algorithms. Swarm intelligence-based optimization algorithms mimic the intelligent behaviour of a group of species in nature. Some state-of-the-art and well-known swarm intelligence-based algorithms are particle swarm optimization (PSO), ant colony optimization and the artificial bee colony algorithm. Evolutionary algorithms simulate biological evolutionary behaviours. The genetic algorithm (GA), differential evolution and evolutionary programming are well-known algorithms in this category. The performance of any such algorithms may be similar across several kinds of application. However, in a larger instant, the performance of any algorithm depends on the associated variables and constraints of the applications (Wolpert and Macready 1997). Hence, researchers have tried to assess the performance of such algorithms by implementing them on various applications on a common platform.

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Scientific law-based algorithms are inspired by the different laws of physics or chemistry. During the past decade, researchers have proposed many algorithms inspired by such laws. A few such algorithms developed during the initial stage are big bang–big crunch (Erol and Eksin 2006), central force optimization (Formato 2008), gravitational search algorithm (GSA) (Rashedi, Nezamabadi-Pour, and Saryazdi 2009), black hole (BH) optimization (Hatamlou 2013) and ray optimization (RO) (Kaveh and Khayatazad 2012). These algorithms have been applied successfully to various real-life optimization problems. Over time, modified and hybrid versions of these algorithms have also been proposed to enhance their performance. The performance of such modified/hybrid versions compared with the basic algorithm has also been investigated and presented by researchers.

Apart from the above, many other scientific law-inspired algorithms have been proposed during recent decades. Some of these algorithms that have caught the attention of the research community are colliding bodies optimization (CBO) (Kaveh and Mahdavi 2014), multi-verse optimizer (MVO) (Mirjalili, Mirjalili, and Hatamlou 2016), electromagnetic field optimization (EFO) (Abedinpourshotorban *et al.* 2016), electro-search (Tabari and Ahmad 2017), heat transfer search (HTS) (Patel and Savsani 2015), optics-inspired optimization (Kashan 2015), atom search optimization (ASO) (Zhao, Wang, and Zhang 2019), equilibrium optimizer (EO) (Faramarzi *et al.* 2020), Henry gas solubility optimization (HGSO) (Hashim *et al.* 2019), arithmetic optimization (Abualigah *et al.* 2021), atomic orbital search (Azizi 2021) and electron radar search (Rahmanzadeh and Pishvaee 2019), for global optimization. These algorithms have undergone experiments on various benchmark functions to assess their performance. A brief overview of some of these algorithms is given in the survey article of Can and Alatas (2015). Such algorithms have also been implemented on the optimization of different engineering applications, such as planar structures (Gholizadeh, Razavi, and Shojaei 2019), distribution networks (Mohamadi, Abedini, and Rashidi 2020) and maintenance processes (Zhang *et al.* 2020). However, the comparison of algorithms in the category of scientific law-inspired algorithms has not been attempted for real-life engineering applications. Hence, the qualitative and quantitative comparison of these algorithms for any real-life engineering application is not available on common platforms. Based on this fact, the present work aims to compare the performance of recently developed scientific law-inspired algorithms for real-life engineering applications.

The power-generating cycle is an important engineering application. The Stirling cycle is one such cycle, in which a heat engine works to generate power. The Stirling heat engine works on the Stirling cycle, which requires a high-temperature heat source for its operation. Performance enhancement of the heat engine cycle is achieved by optimization of its operating variables within predefined operating and output constraints. Most of the constraints are active at the optimized value of operating variables with linear or nonlinear behaviour. Furthermore, some of the operating variables of the Stirling heat engine cycle are continuous while others are discrete. This behaviour of the operating variables and constraints makes the optimization problem of the Stirling heat engine cycle challenging.

In the past decade, many researchers have attempted the optimization of the Stirling heat engine with metaheuristic algorithms. However, most researchers adopted either the GA or the PSO algorithm, while other researchers used other metaheuristics in their optimization investigations. For instance, Ahmadi *et al.* (2013), Arora, Kaushik, and Kumar (2016), Zare and Tavakolpour-Saleh (2016), Toghyani, Kasaeian, and Ahmadi (2014) and Li, Grosu, and Queiros-Conde (2016) adopted the GA for the optimization of various performance parameters of the Stirling heat engine. Duan *et al.* (2014), Zare and Tavakolpour-Saleh (2019) and Ahmadi, Aghaj, and Nazeri (2013) used PSO for the Stirling heat engine optimization. Patel and colleagues used the HTS algorithm (Patel, Savsani, and Mudgal 2017), teaching–learning-based optimization (Patel 2016) and some swarm intelligence-based algorithms (Patel, Savsani, and Tawhid 2019) for the optimization of the Stirling heat engine. Thus, the literature survey reveals that, apart from the HTS algorithm, no scientific law-inspired metaheuristics have yet been explored for the optimization of the Stirling heat engine.

The present study investigates the performance of scientific law-inspired metaheuristic algorithms developed during the past decade for the optimization of the Stirling heat engine. The objectives of the present work are: (1) to investigate the performance of recently developed scientific law-inspired

algorithms (MVO, BH, HTS, ASO, GSA, EO, CBO, RO, EFO and HGSO) for the optimization of a Stirling heat engine; (2) to analysed the statistical significance of the obtained results and rank the algorithms; (3) to identify the effect of various constraint handling methods on the performance of the algorithms; (4) to evaluate the effectiveness of each constraint handling method with the considered algorithms through different performance measures and statistical analysis; (5) to evaluate the behaviour of the considered algorithms with regard to the sensitivity of the operating constraints; and (6) to estimate the convergence behaviour of the algorithms. These contributions are demonstrated through an engineering application of a Stirling cycle-operated heat engine system.

The remainder of the article is organized as follows. Section 2 includes a brief description of scientific law-inspired algorithms considered in the present work. A description of the system, with objective function formulation and constraints, is given in Section 3. An engineering application with decision variables is presented in Section 4, followed by the results and discussion in Section 5. Finally, conclusions are presented in Section 6.

2. Scientific law-inspired optimization algorithms

A brief description of the scientific law-inspired optimization algorithms developed in the recent past and considered in the present work is covered in this section. The basic principle of each algorithm is described here with appropriate references, so that the reader can obtain more information about the algorithms.

2.1. Multi-verse optimizer

Mirjalili, Mirjalili, and Hatamlou (2016) proposed a multi-verse optimizer (MVO) algorithm. The MVO was inspired by the belief of the existence of white holes, black holes and wormhole in the universe. The behaviour of the black and white holes is opposite to each other, while the wormhole connects the different parts of the universe. The black and white holes perform the exploration, while the wormhole executes the exploitation during the course of optimization.

2.2. Black hole algorithm

Hatamlou (2013) proposed a black hole (BH) algorithm. The BH algorithm is inspired by the black hole phenomenon. In the BH algorithm, the best candidate solution acts as a black hole and the remaining solutions are treated as stars. During optimization, all stars move towards the black hole. When any star moves too close to the black hole, it will be replaced by a new star through random generation.

2.3. Heat transfer search algorithm

Patel and Savsani (2015) proposed a heat transfer search (HTS) algorithm. The HTS algorithm is inspired by the natural phenomenon of thermal equilibrium between a system and its surroundings. Any system establishes thermal equilibrium with its surroundings through heat transfer in the form of conduction, convection and radiation. Likewise, the HTS algorithm is executed in three phases, which balance exploration and exploitation during the optimization.

2.4. Atom search optimization

Zhao, Wang, and Zhang (2019) proposed an atom search optimization (ASO) algorithm. ASO is inspired by basic molecular dynamics. In ASO, each atom represents the candidate solution and the quality of the solution is measured by its mass. Furthermore, atoms attract and repel each other based

on the interatomic distance. Lighter atoms have faster movements and represent exploration, while heavier atoms have small movements and represent exploitation.

2.5. Gravitational search algorithm

Rashedi, Nezamabadi-Pour, and Saryazdi (2009) proposed a gravitational search algorithm (GSA). The GSA is inspired by the law of gravity and mass interaction. In the GSA, the mass of the object represents the solution to the optimization problem. The heavier mass of an object signifies a good solution. A heavy object attracts a lighter mass owing to the gravitational effect, which indicates exploration. Likewise, a heavy object moves very little and represents exploitation.

2.6. Equilibrium optimizer

Faramarzi *et al.* (2020) proposed an equilibrium optimizer (EO) algorithm. EO is inspired by the mass balance and mass interaction of a mixture of gases in a control volume while achieving an equilibrium state. In EO, the concentration of gas represents the quality of the solution. During optimization, the concentration of gases is updated based on the equilibrium concentration, generation rate and concentration difference.

2.7. Colliding bodies optimization

Kaveh and Mahdavi (2014) proposed the colliding bodies optimization (CBO) algorithm. CBO is inspired by the movement of the objects due to collisions in an isolated system. The system is composed of stationary and moving objects. Moving objects collide with each other and move at different velocities to approach the stationary objects. This phenomenon improves the position of a moving object and also pushes a stationary object towards a better position.

2.8. Ray optimization

Kaveh and Khayatazad (2012) proposed the ray optimization (RO) algorithm. RO is inspired by the behaviour of rays when they transit from one medium to another. In the RO algorithm, the position of light rays indicates candidate solutions. The path of light rays changes owing to refraction when they travel from one medium to another. Based on the index of refraction, the algorithm balances the exploration and exploitation during the search process.

2.9. Electromagnetic field optimization

Abedinpourshotorban *et al.* (2016) proposed the electromagnetic field optimization (EFO) algorithm. EFO is inspired by the behaviour of electromagnets having different polarities. In EFO, the electromagnetic particles are the candidate solutions. The electromagnetic particles attract or repel each other based on the polarities and move towards the global solution. The ratio between attractive and repulsive forces balance the exploration and exploitation.

2.10. Henry gas solubility optimization

Hashim *et al.* (2019) proposed the Henry gas solubility optimization (HGSO) algorithm. HGSO is inspired by Henry's gas law, which relates to the dissolving of any gas in a liquid. In HGSO, the equilibrium state of the gases is the candidate solution. The gas achieves the highest equilibrium state by updating the solubility and Henry's coefficient. Furthermore, the diversity vector balances the exploration and exploitation during optimization.

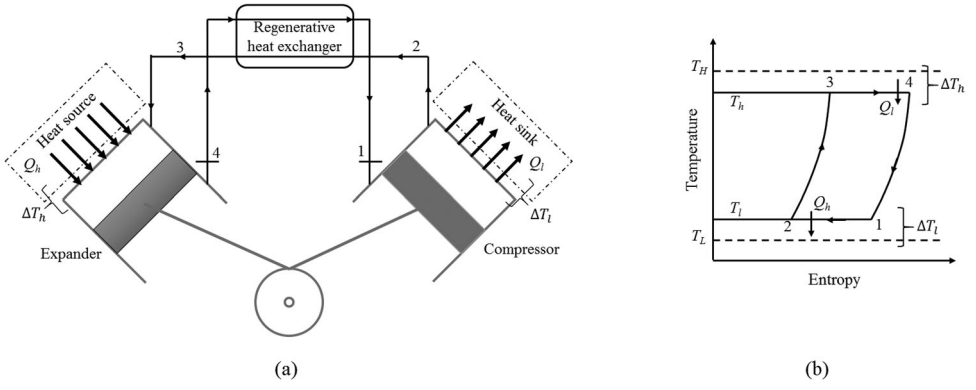


Figure 1. Stirling cycle-based heat engine system: (a) schematic arrangement; (b) thermodynamic presentation.

3. System description, objective function and constraints

A brief description of the Stirling cycle-operated heat engine is covered in this section. Figure 1 shows a line diagram of the Stirling cycle-operated heat engine and its thermodynamic representation. The main components of the system are the compressor, regenerative heat exchanger and expander. The working fluid circulates in a closed loop through these components and produces the required output power through the expander.

The operation of the Stirling cycle-operated heat engine is composed of four processes, as depicted in Figure 1. Of these four processes, two are isothermal and the other two are isochoric. The working gas is compressed isothermally in the compressor during process 1–2 at low temperature, by rejecting heat to the heat sink. The compressed gas performs a constant volume heat recovery (process 2–3) in the regenerative heat exchanger. The high temperature working fluid further expands during process 3–4, where it absorbs heat from the high-temperature heat source and produces the required output power. Finally, the working fluid returns through the regenerative heat exchanger (process 4–1), where it again cools to the low temperature by rejecting heat and is ready for the next cycle. The performance of this heat engine is assessed in terms of a parameter known as thermal efficiency.

The primary objective during the operation of a heat engine is to maximize its thermal efficiency. The thermal efficiency of the Stirling cycle-operated heat engine can be represented mathematically as (Patel, Savsani, and Tawhid 2019):

$$\eta_{\text{th}} = \eta_{\text{c}}\eta_{\pi} \tag{1}$$

where η_{c} is the Carnot efficiency and η_{π} is the second law efficiency, given by

$$\eta_{\text{c}} = 1 - \frac{T_{\text{L}} + \Delta T_{\text{l}}}{T_{\text{H}} - \Delta T_{\text{h}}} \tag{2}$$

$$\eta_{\pi} = \left[1 - \left(\frac{3\mu' \frac{\Delta p}{P_1}}{\eta' \frac{T_{\text{H}} - \Delta T_{\text{h}}}{T_{\text{L}} + \Delta T_{\text{l}}} \log \lambda}\right)\left(\frac{1}{1 + \left(\frac{X}{(\gamma - 1)\log \lambda}\right)\eta_{\text{c}}}\right)\right] \tag{3}$$

The details related to each of the above-mentioned parameters are available in the work reported by Patel, Savsani, and Tawhid (2019).

Operational optimization of the Stirling heat engine which results in maximum thermal efficiency should also satisfy the various output constraints. Different constraint handling methods are explored in the present work to ensure that the output constraints are satisfied. The effectiveness of each of the constraint handling methods, with its statistical significance, is also investigated in this study. The

objective function of the Stirling cycle-operated heat engine is given by

$$\text{Minimize } f(x) = \eta_{\text{th}} + \phi((\bar{g}_j(x))(\bar{h}_i(x))) \tag{4}$$

where the second term indicates the constraint handling method, to take care of the associated equality $(\bar{g}_j(x), j = 1, 2 \ldots n)$ and inequality $(\bar{h}_i(x), i = 1, 2 \ldots m)$ constraints.

The above-mentioned objective function of the system should satisfy the different output constraints described in Sections 3.1 and 3.2.

3.1. Pressure drop constraint

The pressure drop of the system affects the power output of the heat engine. To obtain the desired power output, the pressure drop of the system should not exceed the allowable limit. The pressure drop of the Stirling heat engine system is calculated by

$$\Delta p = \Delta p_{\text{r}} + \Delta p_{\text{f}} + \Delta p_{\text{v}} \tag{5}$$

where Δp_{r}, Δp_{f} and Δp_{v} are the pressure drop due to the regenerator matrix, mechanical friction and piston speed, respectively.

3.2. Exergy efficiency constraint

The energy conversion behaviour of the system can be assessed with the help of exergy efficiency. The exergy efficiency is an indicator of system performance and the operational effect of the system on the surroundings. The exergy efficiency of the system can be calculated by

$$\eta_{\text{ex}} = \frac{\left(\left(1 - \frac{T_0}{T_{\text{H}}}\right) Q_{\text{h}}\right) - \left(\left(1 - \frac{T_0}{T_{\text{L}}}\right) Q_{\text{l}}\right)}{(Q_{\text{h}} - Q_{\text{l}})} \tag{6}$$

where T_0, T_{H} and T_{L} are the temperature of the environment, hot end and cold end, respectively; and Q_{h} and Q_{l} indicate the heat supply and heat rejected, respectively.

4. Engineering application and decision variables

An engineering application of a Stirling cycle-operated heat engine is demonstrated to assess the performance of the scientific law-inspired algorithms considered in the present work. A Stirling heat engine used to produce 13 kW power is operated between a hot-end temperature (T_{H}) of 1300 K and a cold-end temperature (T_{L}) of 288 K. Helium gas is used as a working fluid in the heat engine. The heat engine filled with 0.001135 kg of helium gas during operation. The exergy efficiency of the heat engine should be more than 83% during the operation of the system. The performance optimization (*i.e.* thermal efficiency maximization) of the Stirling heat engine depends on the various operating parameters. In the present work, four operating variables, namely the rotational speed of the engine (N_{r}), mean effective pressure (P_{mean}) of the system, and working fluid temperature differences at the hot end (ΔT_{h}) and cold end (ΔT_{l}), are considered for the optimization. The upper and lower bounds of the operating variables are as follows:

$$1200 \text{ rpm} \leq N_{\text{r}} \leq 3000 \text{ rpm} \tag{7}$$

$$0.69 \text{ MPa} \leq P_{\text{mean}} \leq 6.89 \text{ MPa} \tag{8}$$

$$64.2 \text{ K} \leq \Delta T_{\text{h}} \leq 237.6 \text{ K} \tag{9}$$

$$5 \text{ K} \leq \Delta T_{\text{l}} \leq 25 \text{ K} \tag{10}$$

Table 1. Control parameters of scientific law-inspired algorithms.

Algorithm	Control parameters
MVO	$WEP = 0.2–0.8; TDR = 0–0.85$
BH	Not applicable
HTS	$CDF = 2; RDF = 2; COF = 10$
ASO	$\alpha = 50; \beta = 0.2$
GSA	$G_0 = 100; \alpha = 20$
EO	$a_1 = 2; a_2 = 1$
CBO	Not applicable
RO	$d = 7.5; stoch = 0.35$
EFO	$P_{field} = 0.1; N_{field} = 0.4$
HGSO	$c_1 = 0.1; c_2 = 0.2$

Note: MVO = multi-verse optimizer; BH = black hole; HTS = heat transfer search; ASO = atom search optimization; GSA = gravitational search algorithm; EO = equilibrium optimizer; CBO = colliding bodies optimization; RO = ray optimization; EFO = electromagnetic field optimization; HGSO = Henry gas solubility optimization.

5. Results and discussion

The scientific law-inspired algorithms considered in the present work are implemented with a common platform for the performance optimization (*i.e.* thermal efficiency maximization) of the Stirling cycle-based heat engine system. The common platform is set with a population size of 50 and 100,000 maximum function evaluations. Each algorithm is implemented 100 times to evaluate its optimization and statistical performance. The tuning of control parameters for each algorithm is executed by conducting several trial runs with different permutations and combinations. Fifteen different permutations and combinations of control parameters are considered for each algorithm in this study. The control parameters that give the best average results are selected for the generation of further results and are presented in Table 1.

The comparative results of each algorithm obtained through 100 independent runs are presented in Table 2. The results are presented in the form of the best solution (*i.e.* maximum thermal efficiency), worst solution (*i.e.* minimum thermal efficiency), average solution (*i.e.* average thermal efficiency), standard deviation and success rate of the algorithm. The average computational time required by each algorithm is also mentioned in the comparative results. It can be observed from the results that MVO and GSA produce the identical best solutions and lead to the optimized performance of the Stirling cycle-operated heat engine system. The BH and EFO follow next in terms of the best solution. In terms of average performance (*i.e.* average solution), the BH algorithm performs better than the rest of the algorithms, followed by the EFO and MVO algorithms. Likewise, with 96% success rate, the EFO algorithm performs better than the other algorithms on the success rate comparison, followed by MVO (94%) and EO, GSA and HGSO (93%). The success rate of the rest of the algorithms varies between 88% and 92%. Similarly, the HGSO algorithm produces solutions with less spread and performs better in terms of standard deviation, followed by the EFO and BH algorithms. Thus, the quantitative performance of the considered algorithms varies for different performance criteria. Therefore, to gain more insight into the performance of these algorithms, statistical tests are performed on the results.

The statistical performance of the considered algorithms is evaluated by executing the Friedman test on the results. The statistical analysis also forms the basis for ranking the algorithms. The results of the statistical analysis are presented in Table 3, in the form of Friedman values and normalized values. A higher normalized value indicates the best performing algorithm. A lower normalized value indicate that an algorithm's performance is degraded compared to the best algorithm. The statistical results are shown graphically in Figure 2. It can be observed from the results of Table 3 and Figure 2 that MVO obtains the top rank, followed by the GSA and EFO algorithms.

Table 2. Comparative results of scientific law-inspired algorithms for Stirling cycle-operated heat engine optimization.

Algorithm	Best solution	Worst solution	Average solution	Standard deviation	Success rate	Computation time (minutes)
MVO	0.1551	0.1451	0.1484	2.21E-03	94	23.24
BH	0.1509	0.1434	0.1492	2.12E-03	92	28.13
HTS	0.1506	0.1419	0.1481	2.40E-03	91	27.32
ASO	0.1504	0.1414	0.1467	2.93E-03	89	27.63
GSA	0.1551	0.1428	0.1482	2.79E-03	93	19.78
EO	0.1504	0.1415	0.1476	2.56E-03	93	24.29
CBO	0.1505	0.1425	0.1480	2.47E-03	90	23.18
RO	0.1505	0.1417	0.1481	2.38E-03	88	19.43
EFO	0.1507	0.1432	0.1489	1.82E-03	96	20.45
HGSO	0.1505	0.1444	0.1483	1.69E-03	93	23.57

Note: MVO = multi-verse optimizer; BH = black hole; HTS = heat transfer search; ASO = atom search optimization; GSA = gravitational search algorithm; EO = equilibrium optimizer; CBO = colliding bodies optimization; RO = ray optimization; EFO = electromagnetic field optimization; HGSO = Henry gas solubility optimization.

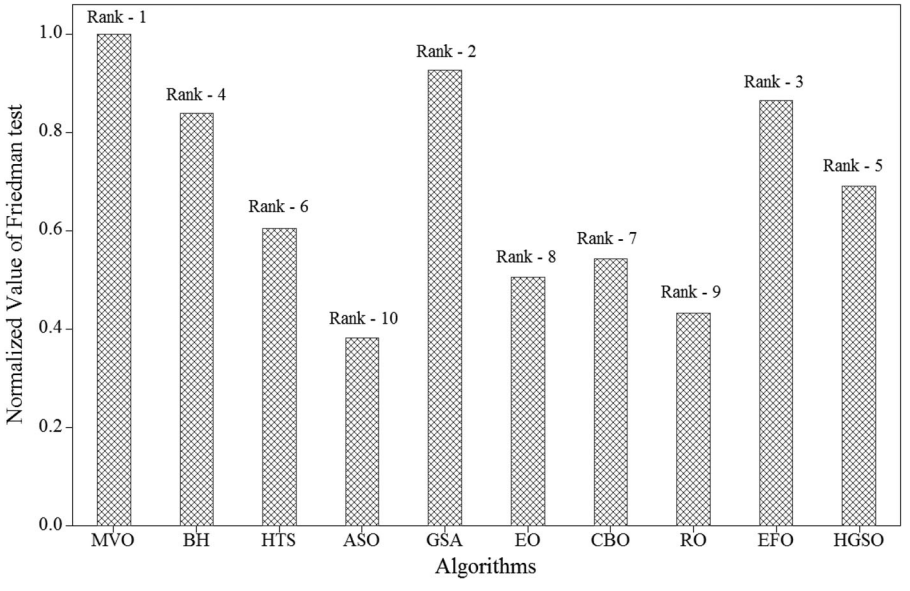


Figure 2. Statistical results and ranking of the algorithms. MVO = multi-verse optimizer; BH = black hole; HTS = heat transfer search; ASO = atom search optimization; GSA = gravitational search algorithm; EO = equilibrium optimizer; CBO = colliding bodies optimization; RO = ray optimization; EFO = electromagnetic field optimization; HGSO = Henry gas solubility optimization.

The considered application of a Stirling cycle-based heat engine system is a constrained optimization problem. The superiority of the feasible solutions method is used to handle the constraints during the performance assessment of the algorithms. The constraint handling methods also affect the performance of any optimization algorithms. Hence, various constraint handling methods, such as static penalty, adaptive penalty, ϵ-constraint and stochastic ranking (Mallipeddi and Suganthan 2010), are used in the experiments to evaluate their effect on the performance of the algorithms. Here, again, the experimental criteria are 100,000 function evaluations and 100 runs of each algorithm are executed with the considered constraint handling method. The comparative results of the proposed investigation are presented in Table 4. The results are presented in the form of the average solution obtained through 100 independent runs of the algorithm along with the standard deviation. It can be observed from the results that the superiority of feasible solutions, ϵ-constraint and stochastic ranking methods perform very similarly with each algorithm for the considered application of a Stirling heat engine. On

Table 3. Statistical comparison of the competitive algorithms.

Algorithm	Freidman value	Normalized value	Rank
MVO	40.5	1	1
BH	34	0.8395	4
HTS	24.5	0.6049	6
ASO	15.5	0.3827	10
GSA	37.5	0.9259	2
EO	20.5	0.5062	8
CBO	22	0.5432	7
RO	17.5	0.4321	9
EFO	35	0.8642	3
HGSO	28	0.6914	5

Note: MVO = multi-verse optimizer; BH = black hole; HTS = heat transfer search; ASO = atom search optimization; GSA = gravitational search algorithm; EO = equilibrium optimizer; CBO = colliding bodies optimization; RO = ray optimization; EFO = electromagnetic field optimization; HGSO = Henry gas solubility optimization.

Table 4. Performance comparison of various constraint handling methods with the considered algorithms.

Algorithm	Static penalty	Adaptive penalty	Stochastic ranking	ϵ-Constraint	Superiority of feasible solutions
MVO	0.1418 (± 3.02E-02)	0.1472 (± 3.13E-02)	0.1483 (± 2.78E-03)	0.1481 (± 2.47E-03)	0.1484 (± 2.21E-03)
BH	0.1425 (± 5.03E-02)	0.1475 (± 9.12E-03)	0.1487 (± 5.19E-03)	0.1489 (± 5.08E-03)	0.1492 (± 2.12E-03)
HTS	0.1411 (± 4.29E-02)	0.1467 (± 4.17E-02)	0.1472 (± 7.84E-03)	0.148 (± 7.63E-03)	0.1481 (± 2.40E-03)
ASO	0.1413 (± 9.36E-03)	0.1452 (± 7.28E-03)	0.1461 (± 5.82E-03)	0.1462 (± 6.38E-03)	0.1467 (± 2.93E-03)
GSA	0.142 (± 3.44E-02)	0.1463 (± 3.73E-02)	0.148 (± 5.17E-03)	0.1477 (± 4.21E-03)	0.1482 (± 2.79E-03)
EO	0.1408 (± 6.91E-02)	0.1458 (± 7.03E-02)	0.147 (± 9.81E-03)	0.1471 (± 7.15E-03)	0.1476 (± 2.56E-03)
CBO	0.141 (± 5.12E-02)	0.1462 (± 9.78E-03)	0.1471 (± 8.14E-03)	0.1478 (± 6.19E-03)	0.148 (± 2.47E-03)
RO	0.1413 (± 2.33E-02)	0.146 (± 8.94E-03)	0.1468 (± 6.73E-03)	0.1481 (± 3.55E-03)	0.1481 (± 2.38E-03)
EFO	0.1421 (± 8.98E-03)	0.1465 (± 6.91E-03)	0.1477 (± 2.16E-03)	0.1483 (± 2.04E-03)	0.1489 (± 1.82E-03)
HGSO	0.1415 (± 9.12E-03)	0.1463 (± 8.79E-03)	0.1469 (± 2.10E-03)	0.148 (± 2.04E-03)	0.1483 (± 1.69E-03)

Note: Data are shown as average solutions (± standard deviation).
MVO = multi-verse optimizer; BH = black hole; HTS = heat transfer search; ASO = atom search optimization; GSA = gravitational search algorithm; EO = equilibrium optimizer; CBO = colliding bodies optimization; RO = ray optimization; EFO = electromagnetic field optimization; HGSO = Henry gas solubility optimization.

the other hand, the performance of MVO and GSA is better than the rest of the algorithms with almost all of the constraint handling methods. However, with the static penalty method, the performance of the EFO and BH algorithms is in line with MVO and GSA.

Statistical analysis of the results is required to observe whether the constraint handling methods produce a significant difference in the output of the algorithms. The Kruskal–Wallis test with Bonferroni *post hoc* process (Ameca-Alducin *et al.* 2018) was performed on the results obtained by each constraint handling method. The results of the test are shown in Table 5. The results are presented in the form of (+) and (−) signs with numbers 1–5, which represent the constraint handling method. The dominance of any constraint handling method over the other methods for a particular algorithm is presented by the (−) sign. Likewise, the (+) sign shows that other constraint handling methods outperform the considered method. It can be observed from the results that stochastic ranking and ϵ-constraint show very similar trends in handling the constraints. Thus, no significant differences are observed in the results produced by these two constraint handling methods with different algorithms. The static penalty and adaptive penalty methods are dominated by the remaining three constraint handling methods for each algorithm. The superiority of feasible solutions method outperforms the remaining four methods for the majority of the algorithms.

The behaviour analysis of each constraint handling method with the considered algorithms was assessed by evaluating the performance measures, namely, average evaluations (AE), success ratio (SR), feasibility ratio (FR) and convergence score (CS). AE shows the average number of evaluations

Table 5. Statistical analysis of results obtained through various constraint handling methods.

Algorithm	Static penalty	Adaptive penalty	Stochastic ranking	ϵ-Constraint	Superiority of feasible solutions
MVO	$2^{(+)}, 3^{(+)}, 4^{(+)}, 5^{(+)}$	$1^{(-)}, 4^{(+)}, 5^{(+)}$	$1^{(-)}, 2^{(-)}, 5^{(+)}$	$1^{(-)}, 2^{(-)}$	$1^{(-)}, 2^{(-)}, 3^{(-)}$
BH	$2^{(+)}, 3^{(+)}, 4^{(+)}, 5^{(+)}$	$1^{(-)}, 3^{(+)}, 4^{(+)}, 5^{(+)}$	$1^{(-)}, 2^{(-)}, 5^{(+)}$	$1^{(-)}, 2^{(-)}$	$1^{(-)}, 2^{(-)}, 3^{(-)}$
HTS	$3^{(+)}, 4^{(+)}, 5^{(+)}$	$3^{(+)}, 4^{(+)}, 5^{(+)}$	$1^{(-)}, 2^{(-)}$	$1^{(-)}, 2^{(-)}, 5^{(+)}$	$1^{(-)}, 2^{(-)}, 4^{(-)}$
ASO	$3^{(+)}, 4^{(+)}, 5^{(+)}$	$3^{(+)}, 5^{(+)}$	$1^{(-)}, 2^{(-)}$	$1^{(-)}, 2^{(-)}, 5^{(+)}$	$1^{(-)}, 2^{(-)}, 4^{(-)}$
GSA	$3^{(+)}, 4^{(+)}, 5^{(+)}$	$3^{(+)}, 4^{(+)}, 5^{(+)}$	$1^{(-)}, 2^{(-)}, 4^{(+)}, 5^{(+)}$	$1^{(-)}, 2^{(-)}, 3^{(-)}$	$1^{(-)}, 2^{(-)}, 3^{(-)}$
EO	$3^{(+)}, 4^{(+)}, 5^{(+)}$	$3^{(+)}, 4^{(+)}, 5^{(+)}$	$1^{(-)}, 2^{(-)}, 4^{(+)}, 5^{(+)}$	$1^{(-)}, 2^{(-)}, 3^{(-)}, 5^{(+)}$	$1^{(-)}, 2^{(-)}, 3^{(-)}, 4^{(-)}$
CBO	$3^{(+)}, 4^{(+)}, 5^{(+)}$	$3^{(+)}, 4^{(+)}, 5^{(+)}$	$1^{(-)}, 2^{(-)}$	$1^{(-)}, 2^{(-)}$	$1^{(-)}, 2^{(-)}$
RO	$2^{(+)}, 3^{(+)}, 4^{(+)}, 5^{(+)}$	$1^{(-)}, 3^{(+)}, 4^{(+)}, 5^{(+)}$	$1^{(-)}, 2^{(-)}, 4^{(+)}, 5^{(+)}$	$1^{(-)}, 2^{(-)}, 3^{(+)}$	$1^{(-)}, 2^{(-)}, 3^{(+)}$
EFO	$2^{(+)}, 3^{(+)}, 4^{(+)}, 5^{(+)}$	$1^{(-)}, 3^{(+)}, 5^{(+)}$	$1^{(-)}, 2^{(-)}$	$1^{(-)}, 2^{(-)}$	$1^{(-)}, 2^{(-)}$
HGSO	$3^{(+)}, 4^{(+)}, 5^{(+)}$	$3^{(+)}, 4^{(+)}, 5^{(+)}$	$1^{(-)}, 2^{(-)}, 4^{(+)}, 5^{(+)}$	$1^{(-)}, 2^{(-)}, 5^{(+)}$	$1^{(-)}, 2^{(-)}, 4^{(-)}$

Note: A minus (−) sign indicates the dominance of the considered constraint handling method over the other methods; a plus (+) sign indicates that the other constraint handling methods outperform the considered method. 1 = static penalty; 2 = adaptive penalty; 3 = stochastic ranking; 4 = ϵ-constraint; 5 = superiority of feasible solutions.
MVO = multi-verse optimizer; BH = black hole; HTS = heat transfer search; ASO = atom search optimization; GSA = gravitational search algorithm; EO = equilibrium optimizer; CBO = colliding bodies optimization; RO = ray optimization; EFO = electromagnetic field optimization; HGSO = Henry gas solubility optimization.

required to obtain an optimum solution. SR indicates the percentage of successful runs, while FR measures the percentage of the feasible run during which the optimum solution is obtained. CS measures the speed and reliability of the algorithms. Detailed formulations of each of these measures are available in the work reported by Nguyen and Yao (2012). The results of the behaviour analysis for each constraint handling method with the considered algorithms are presented in Table 6. It can be observed from the results that based on AE, SR and FR, superiority of feasible solutions performs better than the other methods for each considered algorithm. The ϵ-constraint and stochastic ranking methods perform similarly for all four criteria. Likewise, the performance of these three constraint handling methods with any of the considered algorithms is better than the adaptive and static penalty methods for all four criteria.

The optimized operating variables of the Stirling heat engine, along with the thermal efficiency and output constraints obtained using the MVO, are presented in Table 7. The results show that both of the output constraints are satisfied in the optimized system. Figure 3 shows the convergence behaviour of the algorithms considered here during operational optimization of the Stirling cycle-based heat engine system. The detailed convergence behaviour of each competitive algorithm at the intervals of 10,000, 20,000, 30,000 and 40,000 function evaluations is shown in Figure 4. It can be observed from Figures 3 and 4 that the BH algorithm converges within about 25,000–27,000 function evaluations. The MVO and EFO algorithms converge within about 28,000–32,000 function evaluations. The rest of the algorithms converge within about 35,000–45,000 maximum function evaluations.

Finally, the behaviour of the considered scientific law-inspired algorithms is evaluated with regard to the sensitivity of the operating constraints. The pressure drop and exergy efficiency of the system are the operating constraints of the Stirling heat engine system. Three different values of pressure drop (*i.e.* 24, 25 and 26 kPa) as well as exergy efficiency (*i.e.* 0.82, 0.83 and 0.84) are considered for the investigation. The considered algorithms were tested with each constraint. The average results obtained through 100 independent runs of each algorithm are shown in Table 8 (for pressure drop) and Table 9 (for exergy efficiency). The results are presented in the form of average solution and associated standard deviation. It can be observed from the results that the responses of all of the algorithms are in line with their initial performance for each operating constraint variation.

6. Conclusion

In this study, the performance of scientific law-inspired algorithms developed during the recent past is investigated for the constraint engineering application of a Stirling heat engine system. Ten scientific

Table 6. Behaviour analysis of constraint handling methods with the considered algorithms.

		MVO	BH	HTS	ASO	GSA	EO	CBO	RO	EFO	HGSO
Superiority of feasible solutions	AE	26,843.32 (±1437.43)	23,047.83 (±976.37)	43,793.13 (±1841.93)	39,856.13 (±1874.39)	33,189.63 (±1813.92)	29,064.41 (±1302.57)	40,032.97 (±1247.44)	20,414.68 (±1003,86)	29,768.36 (±1542.17)	22,794.18 (±1053.17)
	SR	0.94 (±0.02)	0.92 (±0.03)	0.91 (±0.03)	0.89 (±0.02)	0.93 (±0.03)	0.93 (±0.04)	0.90 (±0.04)	0.88 (±0.05)	0.96 (±0.04)	0.93 (±0.02)
	FR	0.99 (±0.01)	0.98 (±0.02)	0.99 (±0.02)	0.98 (±0.03)	1 (±0.00)	0.99 (±0.00)	0.98 (±0.02)	0.97 (±0.01)	1 (±0.00)	1 (±0.00)
	CS	28,129.21	24,673.81	46,801.32	42,319.57	36,489.37	31,937.33	41,508.63	21,638.62	31,264.48	24,614.36
ϵ-Constraint	AE	27,143.76 (±1613.44)	23,863.82 (±843.02)	44,031.64 (±2013.28)	40,276.27 (±1763.46)	33,821.69 (±1397.31)	29,661.87 (±1412.53)	40,274.96 (±1903.45)	21,163.19 (±1327.28)	30,594.25 (±1773.81)	23,896.36 (±1449.37)
	SR	0.92 (±0.03)	0.90 (±0.03)	0.88 (±0.05)	0.87 (±0.04)	0.91 (±0.03)	0.91 (±0.02)	0.87 (±0.03)	0.88 (±0.04)	0.93 (±0.02)	0.91 (±0.04)
	FR	0.98 (±0.01)	0.98 (±0.01)	0.97 (±0.02)	0.97 (±0.01)	0.98 (±0.02)	0.98 (±0.03)	0.96 (±0.01)	0.97 (±0.02)	1 (±0.00)	0.99 (±0.00)
	CS	28,732.87	24,902.87	46,957.23	42,773.98	36,851.34	32,488.45	41,932.78	22,087.19	31,764.49	25,032.12
Stochastic ranking	AE	27,435.76 (±1692.66)	23,592.64 (±983.52)	43,987.04 (±1897.31)	40,762.39 (±1492.84)	33,557.16 (±1197.73)	29,812.76 (±1519.22)	40,293.16 (±1879.11)	20,984.29 (±1092.47)	30,721.62 (±1696.23)	23,103.49 (±1328.57)
	SR	0.92 (±0.04)	0.91 (±0.04)	0.90 (±0.04)	0.85 (±0.05)	0.90 (±0.04)	0.91 (±0.03)	0.89 (±0.02)	0.86 (±0.05)	0.91 (±0.02)	0.91 (±0.03)
	FR	0.98 (±0.01)	0.99 (±0.00)	0.98 (±0.00)	0.99 (±0.01)	0.99 (±0.00)	0.98 (±0.02)	0.98 (±0.03)	0.97 (±0.00)	0.99 (±0.00)	0.99 (±0.01)
	CS	29,035.12	24,989.65	47,167.03	42,674.19	36,767.49	32,394.72	41,978.41	21,993.47	31,939.06	24,917.59
Adaptive penalty	AE	30,024.62 (±1907.46)	28,652.28 (±1796.07)	46,287.16 (±1936.38)	41,674.39 (±1812.49)	35,107.38 (±1685.27)	31,237.94 (±1830.28)	42,176.38 (±2018.49)	22,293.72 (±1379.36)	31,754.83 (±1835.56)	24,693.14 (±1670.36)
	SR	0.89 (±0.05)	0.86 (±0.07)	0.87 (±0.05)	0.83 (±0.08)	0.85 (±0.09)	0.86 (±0.06)	0.84 (±0.05)	0.82 (±0.07)	0.86 (±0.10)	0.85 (±0.13)
	FR	0.96 (±0.02)	0.95 (±0.04)	0.99 (±0.00)	0.97 (±0.02)	0.95 (±0.04)	0.95 (±0.02)	0.94 (±0.03)	0.94 (±0.02)	0.95 (±0.05)	0.97 (±0.04)
	CS	32,032.16	32,654.39	50,023.73	44,187.93	37,912.24	34,289.85	43,732.69	23,876.18	33,205.37	26,138.28
Static penalty	AE	31,123.83 (±2094.28)	30,153.72 (±1946.38)	49,038.68 (±2137.19)	42,809.17 (±1963.88)	36,347.28 (±1793.42)	32,198.46 (±2103.19)	43,089.17 (±2103.58)	23,187.37 (±1418.93)	32,364.68 (±1903.17)	25,386.49 (±1703.43)
	SR	0.86 (±0.10)	0.83 (±0.11)	0.83 (±0.09)	0.81 (±0.12)	0.82 (±0.13)	0.83 (±0.05)	0.82 (±0.06)	0.78 (±0.14)	0.81 (±0.09)	0.79 (±0.12)
	FR	0.94 (±0.06)	0.94 (±0.06)	0.96 (±0.01)	0.96 (±0.00)	0.94 (±0.04)	0.93 (±0.04)	0.91 (±0.06)	0.91 (±0.02)	0.92 (±0.03)	0.95 (±0.05)
	CS	33,198.64	33,876.92	51,132.46	44,937.75	38,287.96	34,805.58	44,032.17	24,296.14	34,109.25	27,289.42

Note: MVO = multi-verse optimizer; BH = black hole; HTS = heat transfer search; ASO = atom search optimization; GSA = gravitational search algorithm; EO = equilibrium optimizer; CBO = colliding bodies optimization; RO = ray optimization; EFO = electromagnetic field optimization; HGSO = Henry gas solubility optimization; AE = average evaluations; SR = success ratio; FR = feasibility ratio; CS = convergence score.

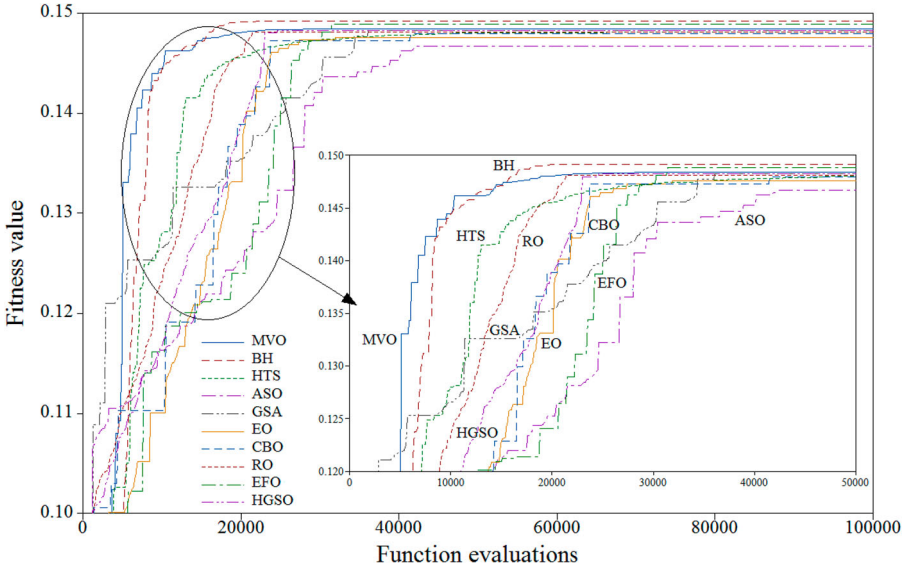


Figure 3. Overall convergence comparison of the considered scientific law-inspired algorithms. MVO = multi-verse optimizer; BH = black hole; HTS = heat transfer search; ASO = atom search optimization; GSA = gravitational search algorithm; EO = equilibrium optimizer; CBO = colliding bodies optimization; RO = ray optimization; EFO = electromagnetic field optimization; HGSO = Henry gas solubility optimization.

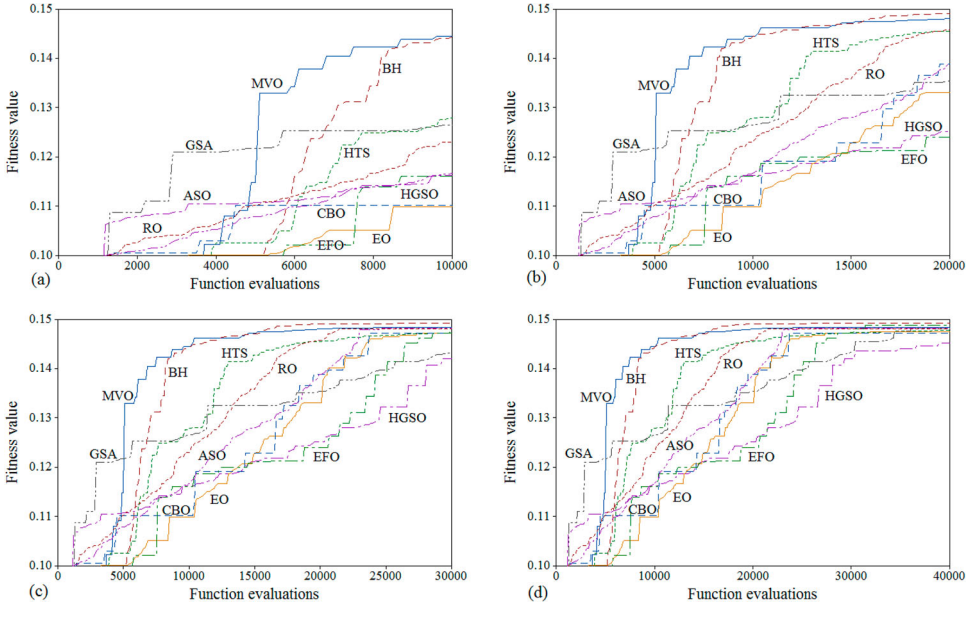


Figure 4. Detail convergence behaviour of each algorithm at intervals of (a) 10,000, (b) 20,000, (c) 30,000, and (d) 40,000 function evaluations. MVO = multi-verse optimizer; BH = black hole; HTS = heat transfer search; ASO = atom search optimization; GSA = gravitational search algorithm; EO = equilibrium optimizer; CBO = colliding bodies optimization; RO = ray optimization; EFO = electromagnetic field optimization; HGSO = Henry gas solubility optimization.

Table 7. Optimized operating conditions of the Stirling heat engine.

Process parameters	Optimized value
Rotation speed of an engine, N_r (rpm)	2947
Mean effective pressure, P_{mean} (MPa)	2.95
Working fluid temperature difference at the hot side, ΔT_h (K)	89
Working fluid temperature difference at the cold side, ΔT_l (K)	5
Exergy efficiency, η_{ex}	83%
Pressure drop, Δp (kPa)	25
Thermal efficiency, η_{th}	0.1551

Table 8. Behaviour of the considered algorithms in response to pressure drop constraints.

Algorithm	Δp = 24 kPa	Δp = 25 kPa	Δp = 26 kPa
MVO	0.1147 (± 9.34E-03)	0.1484 (± 2.21E-03)	0.1839 (± 7.18E-03)
BH	0.1156 (± 7.35E-03)	0.1492 (± 2.12E-03)	0.1847 (± 7.39E-03)
HTS	0.1144 (± 7.63E-03)	0.1481 (± 2.40E-03)	0.1841 (± 6.53E-03)
ASO	0.1131 (± 6.82E-03)	0.1467 (± 2.93E-03)	0.1826 (± 6.28E-03)
GSA	0.1145 (± 6.27E-03)	0.1482 (± 2.79E-03)	0.1842 (± 8.13E-03)
EO	0.1138 (± 7.19E-03)	0.1476 (± 2.56E-03)	0.1833 (± 7.62E-03)
CBO	0.1142 (± 8.03E-03)	0.1480 (± 2.47E-03)	0.1838 (± 6.23E-03)
RO	0.1140 (± 7.46E-03)	0.1481 (± 2.38E-03)	0.1442 (± 6.04E-03)
EFO	0.1137 (± 5.84E-03)	0.1484 (± 1.82E-03)	0.1837 (± 5.97E-03)
HGSO	0.1142 (± 6.13E-03)	0.1484 (± 1.69E-03)	0.1841 (± 5.91E-03)

Note: MVO = multi-verse optimizer; BH = black hole; HTS = heat transfer search; ASO = atom search optimization; GSA = gravitational search algorithm; EO = equilibrium optimizer; CBO = colliding bodies optimization; RO = ray optimization; EFO = electromagnetic field optimization; HGSO = Henry gas solubility optimization.

Table 9. Behaviour of the considered algorithms in response to exergy efficiency constraints.

Algorithm	η_{ex} = 0.82	η_{ex} = 0.83	η_{ex} = 0.84
MVO	0.1796 (± 5.37E-03)	0.1484 (± 2.21E-03)	0.1179 (± 6.79E-03)
BH	0.1807 (± 5.96E-03)	0.1492 (± 2.12E-03)	0.1189 (± 6.14E-03)
HTS	0.1802 (± 4.83E-03)	0.1481 (± 2.40E-03)	0.1177 (± 5.84E-03)
ASO	0.1779 (± 6.06E-03)	0.1467 (± 2.93E-03)	0.1149 (± 7.13E-03)
GSA	0.1794 (± 4.11E-03)	0.1482 (± 2.79E-03)	0.1173 (± 5.18E-03)
EO	0.1785 (± 5.70E-03)	0.1476 (± 2.56E-03)	0.1153 (± 6.37E-03)
CBO	0.1794 (± 6.18E-03)	0.1480 (± 2.47E-03)	0.1182 (± 6.29E-03)
RO	0.1800 (± 5.47E-03)	0.1481 (± 2.38E-03)	0.1184 (± 5.91E-03)
EFO	0.1791 (± 3.08E-03)	0.1484 (± 1.82E-03)	0.1180 (± 3.29E-03)
HGSO	0.1795 (± 3.64E-03)	0.1484 (± 1.69E-03)	0.1178 (± 2.97E-03)

Note: MVO = multi-verse optimizer; BH = black hole; HTS = heat transfer search; ASO = atom search optimization; GSA = gravitational search algorithm; EO = equilibrium optimizer; CBO = colliding bodies optimization; RO = ray optimization; EFO = electromagnetic field optimization; HGSO = Henry gas solubility optimization.

law-based algorithms—MVO, BH, HTS, ASO, GSA, EO, CBO, RO, EFO and HGSO—are considered in the present work. The quantitative and qualitative results of the competitive algorithms are presented with statistical data. Based on the results, the MVO algorithm obtained the top rank, followed by the GSA and EFO algorithms. The effects of five different constraint handling methods on the performance of the considered algorithms are evaluated. Four different measures, namely average evaluations, success ratio, feasibility ratio and convergence score, are used with statistical analysis to assess the performance and behaviour of the constraint handling methods with the considered algorithms. The comparative results indicate that the superiority of feasible solutions method outperforms the remaining four methods for most of the algorithms. Looking at the convergence behaviour, the BH algorithm converges more rapidly to the optimum value than the other competitive algorithms. The response of the considered algorithms is in line with their initial performance for each operating constraint variation. The considered scientific law-inspired algorithms could also be used to optimize other engineering applications.

Disclosure statement

No potential conflict of interest was reported by the authors.

Data availability statement

Data sharing is not applicable to this article as no new data were created or analysed in this study.

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