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Neighborhood Outer Connected Domination in Graphs

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Abstract:	A dominating set S of a connected graph G is called a neighborhood outer connected dominating set, abbreviated as nocd-set, if the induced subgraph $<N(V - S)>$ is connected. The minimum cardinality of a nocd-set of G is called the neighborhood outer connected domination number of G and denoted by $\gamma_{n \textunderscore oc}(G).$ In this paper, we presented the notion of neighborhood outer connected domination in graphs. Further, we calculated the neighborhood outer connected domination number of path, cycle graphs and handle the upper bound for neighborhood outer connected domination number of a graph in terms of the minimum degree among all its vertices. We characterized that G is a connected graph with the neighborhood outer connected domination number less than or equal to the minimum degree and the domination number. Also, we gave the upper bound for neighborhood outer connected domination number of a tree in terms of the order of a graph and discussed the relation between the set of all end vertices of T and set of all support vertices of T, $T \in \mathcal{T}$. Moreover, we proved that the upper bounds for neighborhood outer connected domination number as a function of invariants such as number of vertices p, minimum degree δ, diameter d and other parameters can be derived only by this parameter.
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Neighborhood Outer Connected Domination in Graphs

S. Amutha · K. Suriya Prabha · N. Anbazhagan · K. Jeganathan · Ismail Naci Cangul*

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Keywords Connected dominating set · connected domination number · neighborhood outer connected dominating set · neighborhood outer connected domination number

Mathematics Subject Classification (2010) 05C38 · 05C40 · 05C69

Significance of the work

Domination in graphs is one of the most common research areas in graph theory due to many applications of the idea of domination. Because of rich applications of this general idea, many variants of it have also been introduced and studied. In this work, neighborhood outer connected domination studied and several formulae and bounds are given for this domination parameter. As the notion of neighborhood plays an important role, this domination parameter will be useful for many researchers.

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1 Introduction

For notations and graph theory nomenclature, we refer to Haynes et al., [6], which has an excellent treatment of the fundamentals of the domination in graphs. Let $G = (V, E)$ be a graph with vertex set $V = V(G)$ of order $p = |V|$ and edge set $E = E(G)$ of size $q = |E|$. Let v be a vertex in V. We denote the degree of v in G by $d_G(v)$. The maximum and minimum degree among the vertices of G are denoted by $\Delta(G)$ and $\delta(G)$, respectively. For a set $S \subseteq V$, the subgraph induced by S is denoted by $< S >$. A cycle and path on p vertices are denoted by C_p and P_p, respectively. The open neighborhood of v is $N_G(v) = \{u \in V | uv \in E\}$ and the closed neighborhood of v is $N_G[v] = \{v\} \cup N_G(v)$. For a set $S \subseteq V$, its open neighborhood is the set $N_G(S) = \cup_{v \in S} N_G(v)$, and its closed neighborhood is the set $N_G[S] = N_G(S) \cup S$. The set of all end vertices and the set of all supports are denoted by $\omega(T)$ and $\psi(T)$, respectively. A set $S \subseteq V$ in a graph $G = (V, E)$ is called a dominating set if every vertex $v \in V$ is either an element of S or is adjacent to an element of S. The domination number $\gamma(G)$ of a graph G is the minimum cardinality of a dominating set in G.

The concept of outer connected domination in graphs has been scrutinized and particularized in [1]. Arumugam and Sivagnanam conferred the notion of neighborhood connected domination in graphs, [5].

Bounds on neighborhood total domination in graphs was bestowed by Henning and Nader, [7]. Cyman came up with the perception of the outer connected domination number of a graph in [8]. The notion of split domination number of a divisible dominating graphs was debated by Amutha et al. in [4]. The idea of total outer connected domination in trees was probed by Cyman in [9]. Abundant disparate domination parameters were discussed in [2, 3]. Prabha et al. bestowed the idea of neighborhood outer split domination in graphs in [10]. Motivated by this work, in this paper we introduced the concept of neighborhood outer connected domination in graphs.

A dominating set S of a connected graph G is called a neighborhood outer connected dominating set, abbreviated as nocd-set, if the induced subgraph $< N(V - S) >$ is connected. The minimum cardinality of a nocd-set of G is called the neighborhood outer connected domination number of G and is denoted by $\gamma_{n_oc}(G)$. In this paper, we enumerate the exact value of the neighborhood outer connected domination number of certain graphs and the upper bounds on this parameter have been expostulated by numerous theorems.

2 Neighborhood outer connected domination number

In this section, the neighborhood outer connected domination number of path and cycle graphs are explored and the bounds on this parameter are investigated.

Theorem 1 *For the path P_p, $\gamma_{n_oc}(P_p) = \lceil \frac{p}{3} \rceil$.*
Proof *Let $P_p = (v_1, v_2, \cdots, v_p)$. If $p \equiv 0 \pmod 3$, then $S = \{v_j : j = 3k+2,\ k \geq 0\}$ is a nocd-set of P_p. If $p \equiv 1 \pmod 3$, then $S = \{v_j : j = 3k-2,\ k \geq 1\}$ is a nocd-set of P_p and if $p \equiv 2 \pmod 3$, then $S = \{v_j : j = 3k-1, k \geq 1\}$ is a nocd-set of P_p. Hence $\gamma_{n_oc}(P_p) \leq \lceil \frac{p}{3} \rceil$. Further if S is any γ_{n_oc}-set of P_p, then $N(V-S)$ contains all the internal vertices of P_p and hence $|S| \geq \lceil \frac{p}{3} \rceil$. Thus $\gamma_{n_oc}(P_p) = \lceil \frac{p}{3} \rceil$.*

Theorem 2 *For the cycle C_p, $\gamma_{n_oc}(C_p) = \begin{cases} \lceil \frac{p+1}{3} \rceil \ if \ \ p \not\equiv 0 \ (mod\ 3) \\ \lfloor \frac{p+1}{3} \rfloor \ if \ \ p \equiv 0 \ (mod\ 3). \end{cases}$*

Proof Let $C_p = (v_1, v_2, v_3, \cdots, v_p, v_1)$. Then $S = \{v_j : j = 3k+1, k \geq 0\}$ is a nocd-set of C_p and hence $\gamma_{n_oc}(C_p) \leq \begin{cases} \lceil \frac{p+1}{3} \rceil \ if \ \ p \not\equiv 0 \ (mod\ 3) \\ \lfloor \frac{p+1}{3} \rfloor \ if \ \ p \equiv 0 \ (mod\ 3). \end{cases}$

Now, let S be any γ_{n_oc}-set of C_p. Then $< V - S >$ contains at most one isolated vertex and $< N(V-S) >= \begin{cases} C_p \quad if \ \ p \not\equiv 5 \ (mod\ 3) \\ P_{p-1} \ if \ \ p \equiv 5 \ (mod\ 3). \end{cases}$ Hence $|S| \geq \begin{cases} \lceil \frac{p+1}{3} \rceil \ if \ \ p \not\equiv 0 \ (mod\ 3) \\ \lfloor \frac{p+1}{3} \rfloor \ if \ \ p \equiv 0 \ (mod\ 3) \end{cases}$ and the result follows.

Therefore we deduce the following result:

Lemma 1 *In a graph G with $\delta(G) \geq 2$, there is a cycle of length at least $\delta(G)+1$.*

Theorem 3 *If G is a connected graph of order p, then $\gamma_{n_oc}(G) \leq p - \delta(G)$.*

Proof If $\delta(G) \leq 2$, then the result is obvious. Therefore we may assume that $\delta(G) \geq 3$. By Lemma 1, there exists a cycle of length at least $\delta(G)+1$ in G. Let $C = (u_0, u_1, \cdots, u_r, u_0)$ be a shortest cycle in G of length at least $\delta(G)+1$. We claim that the set $S = V(G) - V(C)$ is a neighborhood outer connected dominating set of G. Certainly, $< N(G-S) > = < N(V(C)) >$ is connected. Suppose S is not a dominating set. Then $N_G(u) \cap S = \phi$ for some vertex $u \in V(C)$. Without loss of generality, we may assume that $u = u_0$ and $deg_G(u_0) = t$. Then $N_G(u_0) = \{u_1, u_{j_1}, u_{j_2}, \cdots, u_{j_{t-2}}, u_r\}$, where $1 < j_1 < j_2 < \cdots < j_{t-2} < r$. Now $(u_0, u_{j_1}, u_{j_1+1}, \cdots, u_r, u_0)$ is a cycle of length at least $\delta(G)$ which is shorter than C, a contradiction.

Theorem 4 *If G is a nontrivial connected graph, then $\gamma_{n_oc}(G) \leq (\frac{\delta+1}{\delta})\ \gamma(G)$.*

Proof Let G be a nontrivial connected graph. Then $\delta(G) \geq 1$. Let S be a $\gamma(G)$-set. If S is a nocd-set of G, then $\gamma_{n_oc}(G) = \gamma(G)$ and there is nothing to prove. Hence, we may assume that $< N(V-S) >$ is a disconnected graph. Then there is an isolated vertex in $< N(V-S) >$ and such a vertex is in S. Let Y be the set of all vertices of S which are isolated in $< N(V-S) >$ and let $S_1 = N(Y) \cap S$. Certainly, both Y and S_1 are independent sets in G. For every $v \in Y$, $N(v) \subseteq S_1$, we have $d_{S_1}(v) \geq \delta$. We now define a sequence $v_1, v_2, \cdots, v_j$ of vertices in Y as follows. Let $Y_1 = Y$, $v_1 \in Y_1$ and let $S_2 = S_1 \backslash N(v_1)$. Let $Y_2 = Y \backslash N(N(v_1))$. If $Y_2 \neq \phi$, then let $v_2 \in Y_2$ and $S_3 = S_2 \backslash N(v_2)$. Proceeding in this way, we define the sets Y_i and S_{i+1} inductively as follows. Let $Y_i = Y_{i-1} \backslash N(N(v_{i-1}))$. If $Y_i \neq \phi$, let $v_i \in Y_i$ and $S_{i+1} = S_i \backslash N(v_i)$. Let j be the first integer such that $Y_{j+1} = \phi$ and let $Y^* = \bigcup_{i=1}^{j} \{v_i\}$ and $S_Y = \bigcup_{i=1}^{j} N(v_i)$. Every vertex in Y is adjacent to at least one vertex in S_Y implying that the set $S \cup Y^*$ is a nocd-set of G. We note that $Y^* \subseteq Y$ and $|Y^*| = j$. Further $S_Y \subseteq S$ and therefore $|S| \geq |S_Y| = \sum_{i=1}^{j} d_G(v_i) \geq j\delta$. If $i \neq j$, then $N(v_i) \cap N(v_j) = \phi$, since Y is an independent set. Hence

$$\begin{aligned} \gamma_{n_oc}(G) &\leq |S \cup Y^*| \\ &= |S| + j \leq |S| + \tfrac{|S|}{\delta} \\ &= \tfrac{\delta+1}{\delta}\, \gamma(G). \end{aligned}$$

Theorem 5 *If T is a tree of order $p \geq 2$, then $\gamma_{n_oc}(T) \leq \lfloor \frac{2p}{3} \rfloor$. Moreover, equality holds if $T = P_4$.*

Proof The result is true for $p = 2$. So we may assume that $p \geq 3$ and let S be a minimum neighborhood outer connected dominating set of T. Let us denote by R any component of $< S >$. Since T is a tree, no two vertices of $N(V(T) - S)$ have a common neighbor in R. Hence $|R \cap N(V(T) - S)| \geq 1$. Moreover, S is a dominating set in T. Thus

$$\begin{aligned} |V(T)| &= |V(T) - S| + |S| \\ &\leq |V(T) - S| + 2|V(T) - S| \\ &\leq p - \gamma_{n_oc}(T) + 2p - 2\gamma_{n_oc}(T). \end{aligned}$$

Finally, we have $\gamma_{n_oc}(T) \leq \frac{2}{3}p$, and so $\gamma_{n_oc}(T) \leq \lfloor \frac{2\,p}{3} \rfloor$.

We now define families of trees as follows. For $p \geq 2$, let

$$\mathcal{T}_p = \{T | T \text{is a tree of order } p \text{ such that } \gamma_{n_oc}(T) = \lfloor \frac{2p}{3} \rfloor\}$$

and $\mathcal{T} = \cup_{p \geq 2} \mathcal{T}_p$.

Lemma 2 *If $T \in \mathcal{T}$, then*

$$|\omega(T)| \leq |\psi(T)| + 2$$

where $\omega(T)$ is the set of all end vertices of T and $\psi(T)$ is the set of all support vertices of T.

Proof Let S be a minimum neighborhood outer connected dominating set of a tree T belonging to $\mathcal{T}$. Then for some positive integer p, we have $T \in \mathcal{T}_p$ and $|S| = \lfloor \frac{2p}{3} \rfloor$. Suppose $|\omega(T)| = |\psi(T)| + r$ for $r > 2$. Let us denote the support vertices of T by $\psi_1, \psi_2, \cdots, \psi_m$ and denote the end vertices of T by $\omega_1, \omega_2, \cdots, \omega_m, \omega_{m+1}, \cdots, \omega_{m+r}$ where $\omega_i \in N_T(\psi_i)$, $1 \leq i \leq m$. Notice that $S - \{\omega_{m+1}, \omega_{m+2}, \omega_{m+3}\}$ is a neighborhood outer connected dominating set of a tree $T' = T - \{\omega_{m+1}, \omega_{m+2}, \omega_{m+3}\}$. Hence

$$\gamma_{n_oc}(T') \leq |S| - 3 = \lfloor \frac{2p-9}{3} \rfloor.$$

On the other hand, by Theorem 5, we have $\gamma_{n_oc}(T') \leq \lfloor \frac{2(p-3)}{3} \rfloor$.

Observation 1 *[1] Let G be a graph with leaves. Then there exists a minimum dominating set of G containing all the support vertices of G.*

Theorem 6 *[1] If G is a graph of order p and G has no isolated vertices, then*

$$\gamma(G) \leq \frac{p}{2}$$

with equality if and only if each component of G is the corona of a connected graph or an induced cycle C_4.

Theorem 7 *If G is a graph of order $p \geq 3$, $diam(G) \leq 2$ and $\delta(G) \geq 2$, then*

$$\gamma_{n_oc}(G) \leq \frac{p+1}{2}.$$

Proof If $\delta \geq \frac{p}{2}$, then $\gamma_{n_oc}(G) \leq \frac{p}{2}$. By Theorem 3, we suppose henceforth $\delta \leq \frac{p-1}{2}$. This implies that for $p \geq 5$, G is not complete and $diam(G) = 2$. Let $u \in V(G)$ be a vertex of minimum degree δ, $N(u) = \{u_1, u_2, \cdots, u_\delta\}$ and let $Y = V - N[u]$. Since G is not complete, Y is not empty and since $diam(G) = 2$, every vertex of Y has a neighbour in $N(u)$.

Case 1. Some vertex u_i, has no neighbour in Y, say u_1. Since $|N(u)| = \delta$, u_1 is adjacent to every other vertex of $N(u)$ and no vertex of Y is isolated in $< Y >$. Let S be a γ-set of $< Y >$. By Theorem 6,

$$\begin{aligned} |S| &\leq \tfrac{p-\delta-1}{2} \\ &< \tfrac{p-3}{2}. \end{aligned}$$

The set $S \cup \{u_1\}$ dominates G and since every vertex of Y has a neighbor in $N(u) \backslash \{u_1\}$, $< N(V \backslash (S \cup \{u_1\})) >$ is connected. Hence $S \cup \{u_1\}$ is an nocd-set of G and

$$\gamma_{n_oc}(G) \leq |S| + 1 \leq \tfrac{p-1}{2}.$$

Case 2. Every vertex u_i has a neighbor in Y and one of them, say u_1, has no $N(u)$-external private neighbor.

Subcase 2.1. The set J of isolated vertices of $< Y >$ is non-empty. Let S_1 be a γ-set of $< Y \backslash J >$. By Theorem 6, $|S_1| \leq \frac{|Y \backslash J|}{2}$. Since $|N(u)| = \delta$, every vertex of J is adjacent to every u_i. Hence the set $S_1 \cup \{u, u_1\}$ dominates G. Moreover, since $J \neq \phi$ and u_1 has no $N(u)$-external private neighbor, $< N(V \backslash S_1 \cup \{u, u_1\})) >$ is connected. Thus $S_1 \cup \{u, u_1\}$ is an nocd-set of G and

$$\begin{aligned} \gamma_{n_oc}(G) &\leq \tfrac{|Y \backslash J|}{2} + 2 \\ &\leq \tfrac{p-\delta-|J|+3}{2} \\ &\leq \tfrac{p}{2}. \end{aligned}$$

Subcase 2.2. $< Y >$ has no isolated vertex. Since u_1 has neighbours in Y but no $N(u)$-external private neighbour. We can suppose, without loss of generality, that u_1 and u_2 have a common neighbor z in Y. For $3 \leq i \leq \delta$, let z be a neighbor of u_i in Y (the vertices $z, z_3, \cdots, z_\delta$ are not necessarily distinct). Let H be the graph defined by $V(H) = Y \cup N(u)$ and $E(H) = E(< Y >) \cup \{zu_1, zu_2, z_3u_3, \cdots, z_\delta u_\delta\}$. The graph H admits a minimum dominating set S_2 containing the support vertices $z, z_3, \cdots, z_\delta$ by Observation 1. Since H has no isolated vertex and the component of H containing z is not isomorphic to C_4 nor to a corona, we deduce

$$\begin{aligned} |S_2| &\leq \tfrac{|V(H)|-1}{2} \\ &= \tfrac{p-2}{2} \end{aligned}$$

by Theorem 6. The set $S_2 \cup \{u_1\}$ dominates G and $< N(V \backslash (S_2 \cup \{u_1\})) >$ is connected. Since u_1 has no $N(u)$-external private neighbor,

$$\begin{aligned} \gamma_{n_oc}(G) &\leq |S_2| + 1 \\ &\leq \tfrac{p}{2}. \end{aligned}$$

Case 3. Every vertex u_i has an $N(u)$-external private neighbor. If u_i and u_j are two different vertices of $N(u)$ admitting two respective private neighbors z_i and z_j belonging to different components of $< Y >$, then $d(z_i, z_j) > 2$ in contradiction with $diam(G) = 2$. Therefore all the $N(u)$-external private neighbors of the vertices u_i belong to the same component G_1 of $< Y >$.

Similarly, all the vertices of the other components of $< Y >$, if any, are adjacent to all the vertices u_i. The sets $V(G) \backslash V(G_1)$ and $V(G_1) \cup \{u_i\}$ for $1 \leq i \leq \delta$ are nocd-sets of G. Therefore,

$$\begin{aligned} \gamma_{n_oc}(G) &\leq min\ \{p - |V(G_1)|,\ |V(G_1)| + 1\} \\ &\leq \tfrac{p+1}{2}. \end{aligned}$$

3 Conclusion

The sharp estimations for the neighborhood outer connected domination numbers of paths and cycles are obtained and the upper bounds for neighborhood outer connected domination number of graphs in terms of the minimum degrees among all of its vertices are determined. Also, an upper bound for the neighborhood outer connected domination number of trees is given.

4 Acknowledgement

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replies to referee comments

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Author Explanation to the Reviewer

Title: Neighborhood Outer Connected Domination in Graphs

In this paper, the authors have introduced and studied the domination parameter namely, the neighborhood outer connected domination number of a graph. We have tried to meet all the suggestions and requirements of the referees as explained below. Please inform us if any of these changes is not satisfactory for the referees.

1. Page 1: Abstract: Line 9: Since there are no such characterizations in the paper, Remove "We characterized … the domination number".

Authors' Explanation: Removed the sentence "We characterized … the domination number". Thank you for the valuable suggestion.

2. Page 1: Abstract: Line 15: Remove "that"
Authors' Explanation: Removed the word "that".

3. Page 1: Abstract: Line 16: Replace "as a function" by "in terms"
Authors' Explanation: Replaced as per your suggestion.

4. Page 1: Abstract: Line 17: Remove "diameter d" as there is no bound obtained in the paper in terms of diameter.
Authors' Explanation: Removed "diameter d".

5. Page 1: Abstract: Line 17: Remove "can be derived … only by this parameter"
Authors' Explanation: Removed the sentence "can be derived … only by this parameter". Thank you for the valuable suggestion.

6. Page 2: Significance of the work: Line 2: Remove "connected domination, outer connected domination, neighborhood connected domination and" as they are not studied as main results in this paper.
Authors' Explanation: Removed the sentence "connected domination, outer connected domination, neighborhood connected domination and". Thank you for the valuable suggestion.

7. Page 2: Significance of the work: Line 4: Remove "are" and "replace "related" by "this"
Authors' Explanation: Removed and replaced as per your suggestion.

8. Page 2: Significance of the work: Line 5: Write "parameter" instead of "parameters"
Authors' Explanation: Corrected the word.

9. Page 2: Significance of the work: Line 6: Replace "these" by "this" and write parameter" instead of "parameters"
Authors' Explanation: Replaced as per your suggestion.

10. Page 2: Introduction: Line 6: Remove parenthesis after $\delta(G)$, and put it after respectively

Authors' Explanation: Corrected as per your suggestion.

11. Page 2: Introduction: Line 13: Remove "of vertices"
Authors' Explanation: Removed "of vertices".

12. Page 2: Introduction: Line 19: Remove "Prabha and Amutha … {4, 5}"and also from the reference list as they are not relevant references for this article.
Authors' Explanation: Removed the above said references 4 and 5 from the Introduction and Reference sections.

13. Page 2: Introduction: Line 27: Domination parameters are not studied in reference 12, So remove it here and also from the reference list.
Authors' Explanation: Removed this reference from introduction and reference list.

14. Page 2: Introduction: Line 29: Replace "the authors" by "we"
Authors' Explanation: Replaced the word.

15. Page 3: Introduction: Line Replace "expostulate" by "expostulated"
Authors' Explanation: Replaced the word as per your suggestion.

16. Page 3 : Neighborhood outer connected domination number: Theorem 21 : Is it Theorem 21 or Theorem 2.1? The same question for all Theorems, Lemmas and Observations. Make necessary changes.
Authors' Explanation: Corrected in all Theorems, Lemmas and observations of this paper. We put the numbers 1, 2, 3… as per journal format. Thank you for the valuable suggestion.

17. Page 3: Neighborhood outer connected domination number: Proof of Theorem 22: Line 4: Congruence should be properly written. Is it correct to write "congruent to 5 mod 3"?
Authors' Explanation: Now the theorem is numbered as 2. Yes, it is correct.

18. Page 3: Neighborhood outer connected domination number: Proof of Theorem 23: Line 2: Where is Lemma 2.3 in the paper?
Authors' Explanation: Now the theorem 23 is numbered as 3. Lemma 2.3 is numbered as 1 and cited according to that.

19. Page 3: Neighborhood outer connected domination number: Proof of Theorem 23: Line 3: Replace $\delta(G)$ by $\delta(G)+1$.
Authors' Explanation: Replaced the notation.

20. Page 3: Neighborhood outer connected domination number : Proof of Theorem 23: Line 5: Replace N(G-D) by <N(G-S)>
Authors' Explanation: Replaced N(G-D) by <N(G-S)> as per your suggestion.

21. Page 4: Neighborhood outer connected domination number: Theorem 24: Start the statement as "If G is a nontrivial connected graph, then …"

Authors' Explanation: Now the theorem 24 is numbered as 4. Started the theorem statement as "If G is a nontrivial connected graph, then …". Thank you for the valuable suggestion.

22. Page 4: Neighborhood outer connected domination number: Proof of Theorem 24: Line 1 : Rewrite proof as "Let G be a nontrivial connected graph. Then $\delta(G)\geq 1$. …"
Authors' Explanation: Now the theorem 24 is numbered as 4. As per your suggestion, we've rewritten the sentence.

23. Page 4: Neighborhood outer connected domination number: Proof of Theorem 24: Line 4 : Rewrite "a vertex in S" as "a vertex is in S"
Authors' Explanation: Corrected.

24. Page 4: Neighborhood outer connected domination number: Proof of Theorem 24: Line 9: Replace "If $Y_2\neq\phi$. Let" by "If $Y_2\neq\phi$, then let"
Authors' Explanation: Replaced as per your suggestion.

25. Page 4: Neighborhood outer connected domination number: Proof of Theorem 24: Line 13: Replace X_{j+1} by Y_{j+1}
Authors' Explanation: Replaced the notation.

26. Page 4 : Neighborhood outer connected domination number : Proof of Theorem 24 : Line 16 : What is the guarantee that $N(v_i) \cap N(v_i)=\phi$ for $i\neq j$?
Author's Justification: $N(v_i) \cap N(v_i) = \phi$ for $i\neq j$, since Y is an independent set.

27. Page 4: Neighborhood outer connected domination number: Proof of Theorem 25: Line 6: What is n? Should it be p?
Authors' Explanation: Now the theorem 25 is numbered as 5. n(T) is replaced by "|V(T)|". Thank you for your suggestion.

28. Page 5: Neighborhood outer connected domination number: Proof of Lemma 22: Line 9: Where is Theorem 2.6? If it is theorem 26, it written after this proof.

Authors' Explanation: Now the theorem 25 is numbered as 5 and the lemma 22 is 2. It is theorem 5 and it is located before this lemma 2.

29. Page 5: Neighborhood outer connected domination number: Proof of Lemma 22: Line 9: Why is the inequality written two times?
Authors' Explanation: Now the lemma 22 is numbered as 2. Now we have written one time and deleted one inequality.

30. Page 5: Neighborhood outer connected domination number: Proof of Theorem 27: Line 1: Is it Theorem 2.4? or Should it be Theorem 23?
Authors' Explanation: Now the theorem 27 is numbered as 7 and the theorem 23 is numbered as 3. It is theorem 3.

31. Page 5: Neighborhood outer connected domination number: Proof of Theorem 27: Case 1: Line 3: Where is Theorem 2.9?
Authors' Explanation: It is theorem 6. We changed the theorem number now. Thank you for the valuable suggestion.

32. Page 6: Neighborhood outer connected domination number: Proof of Theorem 27: Subcase 2.1 : Line 2 : Where is Theorem 2.9?
Authors' Explanation: Replaced the theorem number "2.9" by "6".

33. Page 6: Neighborhood outer connected domination number: Proof of Theorem 27: Subcase 2.2: Line 7: Where is Observation 2.8?
Authors' Explanation: Now observation 2.8 is numbered as 1. Now we corrected it as "observation 1".

34. Page 6: Neighborhood outer connected domination number: Proof of Theorem 27: Subcase 2.2 : Line 10 : Replace n(H) by |V(H)|
Authors' Explanation: Replaced the notation.

35. Page 6: Neighborhood outer connected domination number: Proof of Theorem 27: Subcase 2.2: Line 12: Where is Theorem 2.9?
Authors' Explanation: Replaced the theorem number "2.9" by "6".

36. Once the bound is obtained for any parameter, one must study when is the bound sharp? So the graphs for which the bounds studied in this paper has to be mentioned.
Authors' Explanation: In Theorem 5, we mentioned the sharpness of the bound. Thank you so much for your valuable suggestion.

37. Page 7: References: Remove References 4, 5 and 12 as they are nowhere relevant to this article.
Authors' Explanation: Removed the irrelevant references as per your suggestion are as follows:

1. Sridharan N, Amutha S, Rao SB (2013), Induced subgraphs of gamma graphs. Discrete Mathematics Algorithm and Applications 5 (3): 1-5
2. Prabha KS, Amutha S (2018), Edge mean labeling of a regular graphs. International Journal of Mathematics Trends and Technology 53 (5): 343-352
3. Prabha KS, Amutha S (2018), Split domination number of a congruent dominating graphs. International Journal of Pure and Applied Mathematics 119 (12): 14633-14642.

Final remarks

Thank you very much for your valuable comments and suggestions. Really these suggestions have been very helpful to us to improve the quality of this paper.