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Investigating Pre-Service Teachers' Instrumented Action Schemes in the Conceptualization of Algebraic and Graphical Representations of Functions

Fonksiyonların Cebirsel ve Grafiksel Temsillerinin Anlamlandırılması Sürecinde Öğretmen Adaylarının Enstrümanlı Eylem Şemalarının İncelenmesi

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Abstract: This study investigates how preservice teachers represent functions in a technological environment and examines the instrumental genesis processes involved. It focuses on instrumented action schemes for representing functions algebraically and graphicallyand drawing the graphs of functions in paper-and-pencil and dynamic geometry environments, evaluating the contribution of technological tools to mathematics teaching. Six second-year preservice teachers in an elementary mathematics teacher education program at a public university in Türkiye participated. Data were collected through worksheets, interviews, screen and audio recordings and analyzed using content analysis. Recordings were transcribed and checked with written responses. Data were coded, categorized, and organized into themes within the instrumental approach and interpreted according to the research objectives. Findings indicate that instrumented action schemes remained procedural in the paper-and-pencil environment, whereas in the dynamic geometry environment they developed conceptual and interactive schemes. The dynamic environment's visual and interactive features facilitated preservice teachers' understanding of functions and their algebraic and graphical representations. This study clearly demonstrates that preservice teachers' instrumented action schemes are shaped not only by the digital or traditional tools they use but also by their interactions with these tools; understanding the dynamics underlying these interactions highlights that the instrumental approach provides an essential theoretical framework for comprehending the learning process.

Anahtar Kelimeler: Instrumental approach, Instrumented action schemes, Preservice teachers, Functions, Algebraic and graphical representations

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Öz: Bu çalışma, öğretmen adaylarının fonksiyonları teknolojik bir ortamda nasıl temsil ettiklerini ve bu süreçlerde ortaya çıkan enstrümantal oluşum süreçlerini incelemeyi amaçlamaktadır. Araştırma, öğretmen adaylarının fonksiyonları cebirsel ve grafiksel olarak temsil etme ve fonksiyon grafiklerini çizmesüreçlerinde geliştirdikleri enstrümanlı eylem şemalarına odaklanmakta ve teknolojik araçların matematik öğretimine katkısını değerlendirmektedir. Çalışmaya, Türkiye'de bir devlet üniversitesinde ilköğretim matematik öğretmenliği programının ikinci sınıfında öğrenim gören altı öğretmen adayı katılmıştır. Veriler, çalışma yaprakları, görüşmeler ile ekran ve ses kayıtları aracılığıyla toplanmış ve içerik analizi yöntemiyle incelenmiştir. Elde edilen ses kayıtları yazılı yanıtlarla karşılaştırılarak çözümlenmiştir. Veriler kodlanmış, kategorilere ayrılmış ve enstrümantal yaklaşım çerçevesinde temalar hâline getirilerek araştırma amacı doğrultusunda yorumlanmıştır. Sonuçlar, öğretmen adaylarının geliştirdiği enstrümanlı eylem şemalarının, yalnızca kullandıkları dijital veya geleneksel araçlardan değil, bu araçlarla kurdukları etkileşimden beslendiğini; bu etkileşimin ardında yatan dinamikleri kavramada ise enstrümantal oluşum teorisinin, öğrenme süreçlerini anlamak için güçlü bir kuramsal çerçeve olduğunu açıkça ortaya koymaktadır.

Keywords: Enstrümantal oluşum, Enstrümanlı eylem şemalar, Öğretmen adayları, Fonksiyon, Fonksiyonların cebirsel ve grafiksel temsilleri.

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1. INTRODUCTION

The integration of technology into mathematics education, particularly through dynamic geometry software like Cabri-Geometry, Geometer's Sketchpad, and GeoGebra, offers transformative potential for teaching abstract mathematical concepts related to functions by enabling dynamic transitions between multiple representations of functions (algebraic, graphical, numerical) (Hollebrands, 2007; Zbiek & Hollebrands, 2008). Recent studies have further highlighted the transformative role of digital technologies in mathematics education and, in particular, their contributions to supporting visualization, representational transitions, and conceptual understanding in learning contexts related to algebra and functions (Weigand et al., 2024; Haspekian et al., 2024; Zhang et al., 2025). While traditional paper-and-pencil environments limit conceptual exploration, these digital tools facilitate deeper understanding of fundamental, yet challenging concepts related to functions, variables and equality (Kieran, 1992) by concretizing abstract ideas and strengthening representational connections (İlhan & Aslaner, 2020; Trouche, 2005). The effectiveness of such technology integration depends critically on teachers' ability to synthesize their mathematical knowledge with emerging technological competencies (Mishra & Koehler, 2006), a process theoretically framed as instrumental genesis (Verillon & Rabardel, 1995) where tools evolve into cognitive instruments through developing instrumented action schemes (Trouche, 2004). Despite growing recognition of this approach in mathematics didactics (Trouche, 2020), significant gaps remain in understanding preservice teachers' scheme development during transitions between the algebraic and graphical representations of functions in dynamic environments (Haspekian & Artigue, 2007; Turgut & Drijvers, 2021), with most research focusing on student learning rather than teacher preparation (Çetin, 2017).

Although previous studies have emphasized that technology-supported instruction can facilitate the concretization of abstract mathematical ideas and support transitions between multiple representations (Kaput, 1998; Trouche, 2005; Haspekian, 2005; İpek & Baran, 2011), there is still limited understanding of how preservice teachers develop instrumented action schemes while conceptualizing the algebraic and graphical representations of functions. Recent reviews have revealed that digital environments can enhance mathematics learning through dynamic visualization and interactive presentations; but these reviews do not explain in detail how teacher candidates develop instrumental action schemas in different learning environments (Zhang et al., 2025; Weigand et al., 2024). In particular, although a number of studies have examined the effects of technology-supported instruction and the use of dynamic digital tools in mathematics education (Haspekian, 2005; Nwabueze, 2016), studies comparing paper-and-pencil and dynamic geometry environments within the framework of instrumental genesis remain scarce. In addition, much of the existing literature on transitions between multiple representations has focused primarily on learners rather than preservice teachers (Gürbüz & Şahin, 2015; Çetin, 2017; Eroğlu, 2020). However, preservice teachers, as future teachers, may develop a different form of instrumental genesis that involves not only technological tool use but also disciplinary and pedagogical knowledge (Tapan, 2006). Therefore, there is a need for research that examines how preservice teachers' instrumented action schemes are shaped across different learning environments in the conceptualization of functions and their algebraic and graphical representations.

In this context, the present study aims to examine preservice teachers' instrumented action schemes in the process of conceptualizing the algebraic and graphical representations of functions in paper-and-pencil and dynamic geometry environments. The main focus of this study is the analysis of instrumented action schemes developed in paper-and-pencil and dynamic geometry environments, in relation to the algebraic and graphical representations of functions, as well as the identification of similarities and differences between these schemes. Within this framework, the study seeks to answer the following research problem and sub-problems. Research Problem: What are the instrumental genesis processes of pre-service teachers regarding the algebraic and graphical representations of functions in dynamic geometry software and paper-and-pencil-based learning environments?

Research Sub-Problems:

1- What are the instrumented action schemes of pre-service teachers regarding the algebraic and graphical representations of functions in the paper-and-pencil environment?
2- What are the instrumented action schemes of pre-service teachers regarding the algebraic and graphical representations of functions in the dynamic geometry environment?
3- What are the differences between the instrumented action schemes of pre-service teachers regarding the algebraic and graphical representations of functions in paper-and-pencil and dynamic geometry environments?

Framework

This study examines the instrumented action schemes of preservice mathematics teachers in making sense of the algebraic and graphical representations of functions with Cabri-Geometry, emphasizing the role of technology-supported instruction in concretizing abstract concepts and facilitating representational transitions, as highlighted by Kaput (1998). Dynamic geometry software enables preservice teachers to develop instrumented action schemes that integrate the algebraic, verbal, and graphical representations of functions simultaneously (Trouche, 2005). This process highlights the significance of multiple representations in mathematics education. Multiple representations refer to expressing a mathematical concept through various forms (algebraic, graphical, numerical, and verbal) and establishing connections among these representations (Ainsworth, 1999). In particular, dynamic geometry software enhances preservice teachers' ability to express a function algebraically while simultaneously visualizing its graph, thus supporting the development of skills to transition between multiple representations (MacGregor & Stacey, 1993). Recent research has shown that dynamic geometry environments can support students' instrumentalization processes by shaping their actions through manipulation, visualization, and interaction, thereby strengthening visual-based mathematical reasoning (Pittalis & Drijvers, 2023). Research originating in France (Guin & Trouche, 2002) underscores the importance of tool use in algebra instruction and notes that such processes contribute to the development of preservice teachers' abilities to visualize abstract concepts and explore mathematical relationships. Within the instrumental approach, these processes can be understood through instrumental genesis, which explains how an artefact becomes an instrument through two interrelated dimensions: instrumentation and instrumentalization (Vérillon & Rabardel, 1995; Trouche, 2004). Instrumentation refers to the way the tool shapes the user's activity, thinking, and strategies, whereas instrumentalization refers to the way the user adapts, organizes, and transforms the tool according to his or her own mathematical purposes. In the context of this study, these two dimensions are important for understanding how preservice teachers use Cabri-Geometry not only to draw the graphs of functions, but also to interpret the relationships between their algebraic and graphical representations. Accordingly, the instrumented action schemes developed by preservice teachers were examined in relation to how the technological environment both guided their actions and was reshaped by them during the process of conceptualizing functions and their representations. Within the context of this conceptual framework, the methodology designed to address the research problem is presented in the following section.

2. METHOD

This section addresses the research design and its rationale, the structure of the selected model, the research environment and participants, participant selection criteria, the design of instructional materials and lesson plans used during the teaching process, the pilot study and its implementation, the data collection instruments relevant to the research model and problem, and the procedures for data analysis. Additionally, issues related to the validity and reliability of the data collection and analysis processes are discussed.

2.1. Research design

To investigate the instrumental processes of preservice teachers in both computer-based and paper-and-pencil environments, a qualitative research approach was adopted. As described by Frankel and Devers

(2000), this approach allows for flexible data collection in natural environments and supports inductive analysis. The study was implemented as a teaching experiment aimed at revealing preservice teachers' instrumented action schemes related to the algebraic and graphical representations of functions and graph construction. Among the qualitative research methodologies, the teaching experiment involves designing structured learning activities within an exploratory conceptual framework to influence participants' mathematical knowledge structures, with data typically collected through techniques such as clinical interviews, direct observations, field notes, and video recordings (Steffe & Ulrich, 2014; Yenmez, 2022). This study was conducted based on the teaching experiment design. The researcher developed assumptions regarding the participants' conceptual understanding of the topic and designed activities supported by Dynamic Geometry Software (Cabri-Geometry). The instruction was personally implemented by the researcher. Throughout the process, audio and screen recordings, worksheets, and interview data were qualitatively analyzed. The mathematical thinking processes and constructed knowledge of the participants were modeled and subjected to cognitive analysis. The researcher-led instruction, the testing and reconstruction of assumptions about participants thinking based on data, and the adjustment of the learning environment accordingly clearly align the study with the methodology of a teaching experiment.

2.2. Research environment and participants

This study was conducted at Bursa Uludağ University, Faculty of Education, in classrooms equipped with smart boards compatible with the Cabri-Geometry software. In Türkiye, becoming a teacher requires completing high school and obtaining a sufficient score on the Higher Education Institutions Examination (YKS) to enroll in four-year teacher education programs within faculties of education. The participants of this research were second-year preservice teachers in the Mathematics Teacher Education Program who had successfully passed the YKS and had not yet taken any courses in computer-assisted mathematics instruction. This grade level was purposefully selected, as it ensured that the preservice teachers experienced Cabri-Geometry for the first time. Through this experience, they developed their own methods during the construction process, built new mathematical structures, established connections between multiple representations, and more clearly observed the differences between paper-and-pencil and digital environments. During the implementation, participants primarily worked in a lecture-oriented seating arrangement; however, group activities were occasionally organized to increase classroom interaction. While 20 preservice teachers participated in the pilot study, the main study included 34 participants. In the main implementation, alongside classwide activities, six preservice teachers were purposively selected and divided into three pairs to form special study groups. This selection was carried out by the researcher and a field expert based on participants' achievement levels, resulting in three distinct groups: the first group consisted of preservice teachers at a medium achievement level, the second group at a high level, and the third group at a low level. The grouping decisions were informed by data obtained from a three-session Cabri-Geometry training provided prior to the implementation, as well as responses to functions-related questions and open-ended pre-interview forms. Such diversity was essential to examine how instrumental genesis processes take shape across different levels of achievement and to enhance the generalizability of the findings. Finally, semi-structured individual interviews were conducted with the six selected preservice teachers outside regular class hours, on a voluntary basis, and in a quiet classroom setting.

2.3. Data collection tools and procedure

The data collection and analysis process consisted of two phases: a pilot study and the main implementation. The pilot study, conducted in the spring semester of 2022–2023 with 20 preservice mathematics teachers at a state university in Türkiye, focused on functions and their algebraic and graphical representations, dynamic geometry software, and instrumental genesis theory. Based on pilot findings, the instructional materials and data collection tools were revised. In the main study, instrumental genesis processes were examined in both Cabri-Geometry and paper-and-pencil environments using worksheets, written and face-to-face interviews, audio, and screen recordings. These

tools were used to examine pre-service teachers' instrumental action schemes and to investigate the ways in which the technological environment shaped their actions—and was in turn shaped by their actions—during the conceptualization process. The tools were refined using pilot analysis, expert opinions, and participant feedback. The main implementation took place during the fall semester of 2023–2024 in the Computer Assisted Mathematics Instruction course with a different cohort at the same university. The researcher ensured data were collected naturally through active interaction with participants.

Three structured worksheets were designed for both environments, progressing from linear to polynomial functions. Worksheet 1 addressed linear equations (x=a, y=b, y=ax, y=x+b, y=ax+b); Worksheet 2 focused on quadratic functions ($f(x)=ax^2$, ax^2+bx, ax^2+bx+c); and Worksheet 3 covered cubic polynomials ($f(x)=ax^3$, ax^3+bx^2, ax^3+cx, ax^3+d). Preservice teachers completed all worksheets in both environments to compare their instrumented action schemes and conceptual development across different representations.

Audio Recordings: To examine in detail the preservice teachers' use of Cabri-Geometry, six participants were grouped into pairs. Their verbal interactions during lessons were recorded using audio devices. These recordings were used to analyze their interactions and cognitive developments during problem-solving.

Screen Recordings: Screen recording software was used to capture the participants' interactions within the Cabri-Geometry environment and their processes of graph construction. These recordings allowed the researcher to observe participants' usage schemes and the relationships they formed with mathematical structures.

Written Interviews: In this study, a structured interview form was developed to elicit the views of pre-service teachers. The form, consisting of open-ended questions, was validated through expert review and a pilot implementation, after which the necessary revisions were made and the final version was established. The interview form comprised nine questions designed to assess pre-service teachers' cognitive levels regarding functions, multiple representations, and the use of technology.

Face-to-face interviews: The semi-structured interviews conducted after the application aimed to reveal the teacher candidates' experiences with the Cabri-Geometry environment, the challenges they encountered, and the instrumental action schemes they developed throughout the process. Conducted outside of class hours on a voluntary basis in a quiet classroom setting with six teacher candidates, these interviews were designed to assess participants' mathematical thinking processes and examine their conceptualization of functions through algebraic and graphical representations. Additionally, the interviews focused on revealing the teacher candidates' primarily procedure-based and limited approaches in the paper-and-pencil environment, as well as their conceptual contributions, perceptual transformations, and tool-based action plans evolving from technical skills toward conceptual understanding within the Cabri-Geometry environment. In this context, appendix (see Appendix 1) presents the mapping between the research questions, data collection tools, and analysis procedures.

2.4. The role of the researcher

In qualitative research, the researcher plays an active role in data collection (Creswell, 2013). This study was conducted by two mathematics education specialists. One served as a participant observer directly involved in instruction, while the other, an expert in technology-enhanced instruction, supported all stages of the research. The researchers observed participants interactions, provided guidance, and shaped the learning environment. Ongoing analysis after each session enabled iterative adjustments. Data from the pilot study were used to strengthen research reliability and refine data collection tools, deepening the understanding of learning processes.

2.5. Data analysis

The data obtained in this study were analyzed using content analysis and interpreted within the framework of instrumental genesis theory. The analysis process was conducted in several stages. In the first stage, audio recordings collected during classroom activities were transcribed and examined together with participants' worksheets and screen recordings from the Cabri-Geometry environment. In addition, data obtained from written responses and semi-structured interviews with selected participants were analyzed and compared with the existing data, thereby ensuring consistency across sources. Subsequently, all data were coded. During the coding process, particular attention was given to participants' statements regarding the instrumental techniques they employed, their instrumental genesis processes, and the instrumented action schemes they developed. Special attention was also paid to indicators of instrumentation and instrumentalization. Instrumentation was examined through episodes showing how the technological environment shaped participants' actions, strategies, and ways of thinking, whereas instrumentalization was examined through episodes showing how participants adapted, organized, and transformed the available tools according to their own mathematical purposes. This stage yielded a substantial number of codes; however, to avoid fragmentation in reporting, codes with similar content were grouped into categories. These categories were then reorganized in line with the components of instrumental genesis theory (instrumented techniques, theorems-in-action, and concepts-in-action, and indicators related to instrumentation and instrumentalization) and refined into themes. Through this process, the instrumental techniques employed to uncover the instrumental genesis processes and instrumental action schemes developed by pre-service teachers in both paper-and-pencil and dynamic geometry software environments were identified. Finally, the results were interpreted in accordance with the research purpose and the themes that emerged. This multi-stage analytical procedure enabled a comparative examination of the instrumental action schemes developed by pre-service teachers across different learning environments.

2.6. Validity and reliability

In qualitative research, validity and reliability are ensured through transferability, credibility, confirmability, and dependability. To strengthen credibility, the researchers engaged with participants over an extended period, consulted experts during analysis, and documented the process thoroughly. Data were collected voluntarily from preservice teachers, and reliability was supported through direct quotations. Participant anonymity was maintained by removing personal identifiers, ensuring privacy in line with ethical standards.

3. FINDINGS AND INTERPRETATION

This study analyzed the processes of six pre-service mathematics teachers, working in pairs, as they solved algebraic expressions (linear, quadratic, and cubic) and generated corresponding graphs in the Cabri Geometry environment within the framework of instrumental genesis. Worksheets revealed insights into both instrumentation (learning the tool's technical use) and instrumentalization (adapting it for personal purposes), leading to the identification of the action schemes developed throughout the process.

3.1. Instrumented techniques, instrumental genesis process, and instrumented action schemes related to linear equations

The linear equations worksheet *(f(x)=a, f(x)=ax, f(x)=ax+b)* was implemented in both paper-and-pencil and Cabri-Geometry environments. In paper-and-pencil, participants used techniques like point plotting, visual analysis, slope examination, and constant term analysis. Cabri-Geometry enabled dynamic techniques including coordinate visualization, dragging for real values, trace observation, intersection identification, and numerical element integration. While both approaches shared core methods, Cabri-Geometry's interactive features (real-time dragging, trace analysis) enhanced conceptual understanding of linear relationships through direct manipulation and observation of parameter effects.

In the first group (P1–P2), a standard approach was observed in the paper-and-pencil environment. For example, they used the rule-based statement "The line $x = 4$ is a line that passes through the point 4 on the x-axis and is parallel to the y-axis." However, as shown in Figure 1, they faced challenges with Cabri-Geometry, such as the formation of discontinuous traces due to the selection of integer-valued points. P1's comment "No matter what value we assign to y on $x = 4$, x will not change" demonstrated an understanding of the concept of a constant function, while also revealing difficulties with using the dynamic geometry environment. This finding reflects instrumentation, as the software environment shaped the participants' actions and observations.

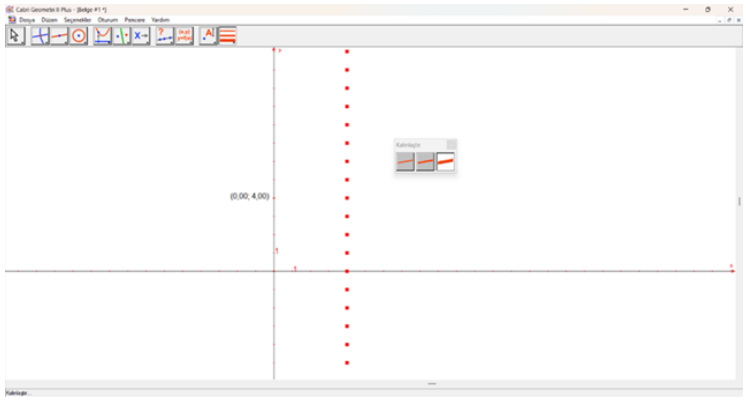


Figure 1. Discontinuous trace

The second group (P3–P4) implemented the theorem "two points determine a line" in the paper-and-pencil environment. However, as seen in Figure 2, they experienced difficulty aligning the coordinate axes orthogonally in Cabri-Geometry. At this stage, the researcher provided guidance on using the axes tool correctly, verifying right angles with the angle measurement tool, and selecting real-valued points. P4's conclusion—"What we learned from this lesson is that a line consists of a set of points"—suggested an emerging understanding of the concept of locus through this guidance. This situation can be interpreted as the beginning of instrumentalization, as participants began to reorganize existing tools in a manner consistent with their mathematical interpretations.

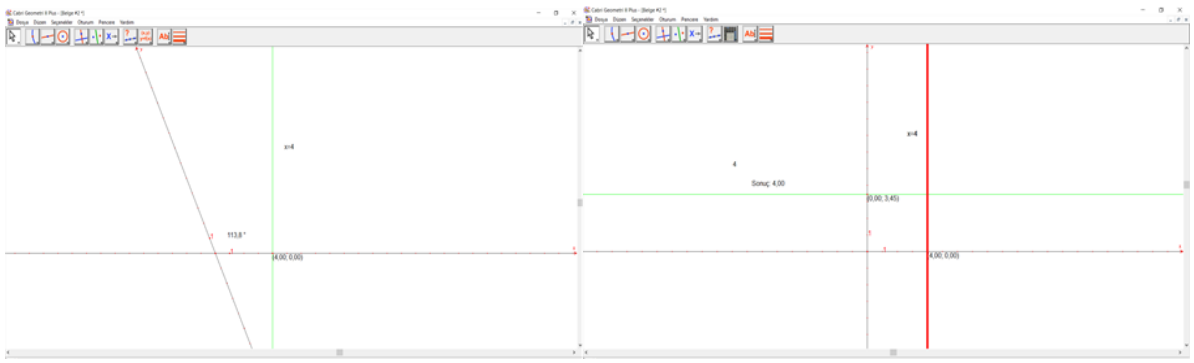


Figure 2. Misaligned coordinate system and corrected axes with line construction

The third group (P5–P6) employed a standard "point plotting and connecting" action scheme in the paper-and-pencil environment (Figure 3).

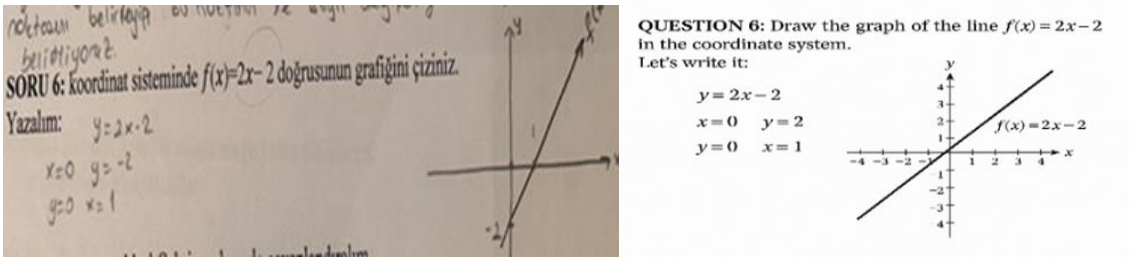


Figure 3. Point plotting and connecting

In Cabri-Geometry, P5's observation—"As we move a, the slope changes"—indicated an evolving understanding of the dynamic nature of parameters (Figure 4). Instrumentation was evident in this group, and P6's solution using the formula (ax) + b showed that they could use the software tools at a basic level. However, the misconception that the coefficient b causes a horizontal shift indicated that their instrumental genesis process had not yet been completed.

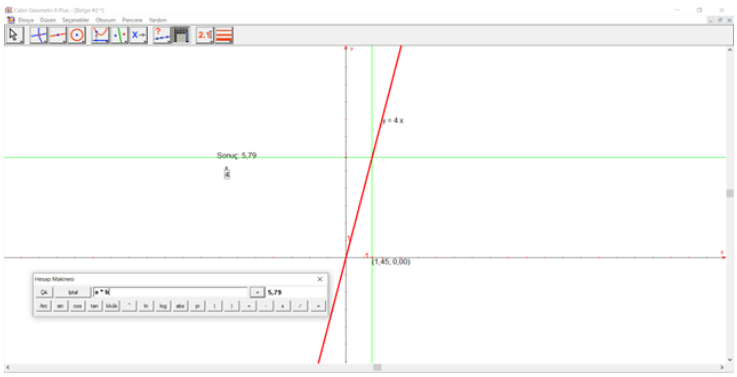


Figure 4. Observation of parameter changes

One of the most significant findings of this study was the transformation of static schemes in the paper-and-pencil environment into dynamic schemes in Cabri-Geometry across all groups. This transformation took place in three primary ways: From assigning values (rule-based) to interactive control of parameters, From connecting plotted points (rule-based) to understanding the geometric locus, From fixed drawing (rule-based) to dynamic trace tracking. On the whole, these findings show that the use of tools and the functionalization of tools become more prominent in the Cabri-Geometry environment compared to the pen-and-paper environment.

3.2. Instrumented techniques, instrumental genesis process, and instrumented action schemes related to quadratic parabolic equations

This worksheet focused on quadratic equations of the forms $f(x) = x^2$, ax^2+bx, ax^2+bx+c, and ax^2+c, implemented in both computer-based and paper-and-pencil environments. Participants were expected to solve these equations and graph them accordingly. The worksheet aimed to support the visual construction of the parabola and its properties through the geometric path of a moving point in the analytic plane. Throughout the process, students employed various techniques based on problem type, often using similar instrumented techniques in both environments. In the paper-and-pencil setting, these included assigning values, calculating outputs, finding roots, identifying the vertex, computing the discriminant, factoring, and analyzing coefficient effects. In Cabri-Geometry, corresponding techniques involved defining coordinates and functions via the calculator tool, transferring measurements, constructing parallel/perpendicular lines, using the trace tool, identifying intersections and loci, and analyzing coefficients on the graph. For example, in the first group's paper-and-pencil work, to solve and graph the function $f(x)=-3x^2+5x$ (as seen in Figure 5), P2 attempted to construct the graph by assigning different values to x and stated, "I used the formula $k = -b/2a$ and found the value 5/6." indicating correct vertex calculation. For the same function, P1 found the roots and noted that the parabola opens downward due to the negative coefficient. However, unlike P2, P1 inferred the vertex position based on

graphical observations. The theorem-in-action applied by the first group was interpreted as follows: "If a parabolic function is to be graphed, points are either calculated by assigning specific x-values or the graph is formed by determining roots and the vertex." This theorem-in-action was found to rely on the concepts of 'effects of function coefficients on the graph' and 'point determination in graphing.'

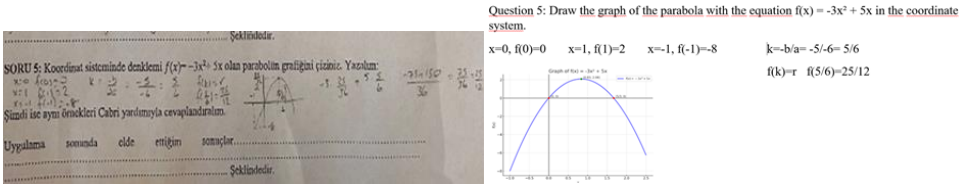


Figure 5. P2's solution to the parabolic equation

Upon transitioning to Cabri-Geometry (as shown in Figure 6), dynamic operations were applied in a planned manner. While drawing the function $f(x)=3x^2$, P1 expressed observations such as, "*As the coefficient a increases, the value of f(x) also increases. Thus, the arms of the parabola approach the y-axis.*" However, instead of drawing the graph, P1 relied on numerical relations, stating, "*Actually, there was no need to draw this; since the result for x^2 is 1, and for $3x^2$ it is 3, by drawing $f(x) = x^2$, we can imagine the graph of $f(x)=3x^2$.*" This suggests that P1 proceeded with intuitive inferences and limited the use of software tools to mathematical relationships. This finding reflects instrumentation, since the software environment shaped the participant's observations of coefficient change and graphical behavior.

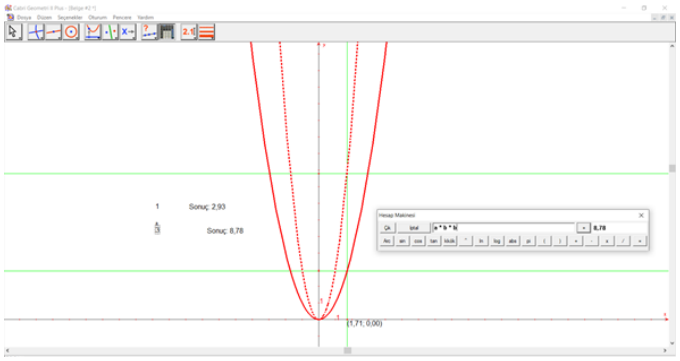


Figure 6. Excerpt from the screen recording of P2's coefficient analysis for quadratic functions worksheet

After adopting the general graphing of parabolic functions, P1 and P2 constructed graphs of $f(x)=2x^2+4$, $f(x)=4x^2-3$, and $f(x)=-3x^2+5$. It was observed that the functional invariant "interpreting the effects of coefficients on the graph," which underpinned their graphing process, transformed into "using dynamic tools to identify the axis of symmetry, vertex, and establish graphical relationships" in the Cabri-Geometry environment. Considering that the visualization and interaction possibilities provided by Cabri-Geometry allowed preservice teachers to develop schemes for interconceptual relations and dynamic tool usage, this process can be regarded as an instrumental genesis process. In the second group's (P3–P4) paper-and-pencil work, the theorem-in-action was identified as: "If the equation of a parabola in the analytic plane is to be solved and its graphical representation constructed, the existing coefficients in the equation are examined for their effects on the graph." This theorem-in-action is based on the application of the methods 'analytic solving' and 'graph drawing' or 'geometric effects of coefficient analysis' (e.g., point determination, discriminant calculation, axis of symmetry identification, and analysis of the parabola's arms' width). As observed in P4's work (Figure 7), "For the function $f(x)=-2x^2+3x-5$, I first found the y-intercept at $(0,-5)$ by assigning $x = 0$. Upon calculating the discriminant, $\Delta = -31$, I understood that the parabola does not intersect the x-axis. I determined the vertex using the formula $-b/2a$ as $(3/4,-31/8)$." These analyses allowed the preservice teachers to draw the graph step-by-step on paper.

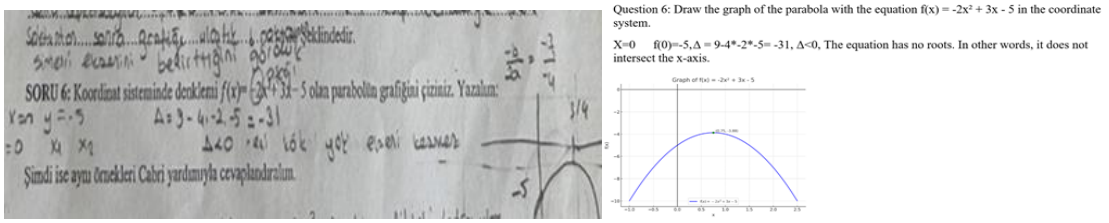


Figure 7. Excerpt from P4's paper showing parabolic solution steps and graph

In the Cabri-Geometry environment, the functional invariant of "assigning values, determining roots, and indicating points in the analytic plane" was found to transform into "dynamic point determination, analyzing the graphical effects of coefficients, and locus construction." P3's comment, "*When I changed the coefficient a from −3 to −2, I observed that the parabola's arms widened. Changing coefficient b shifted the axis of symmetry.*" illustrates this transformation (Figure 8). This transformation may also be interpreted as the beginning of instrumentalization, since the participants started to use the available tools in line with their own mathematical interpretations.

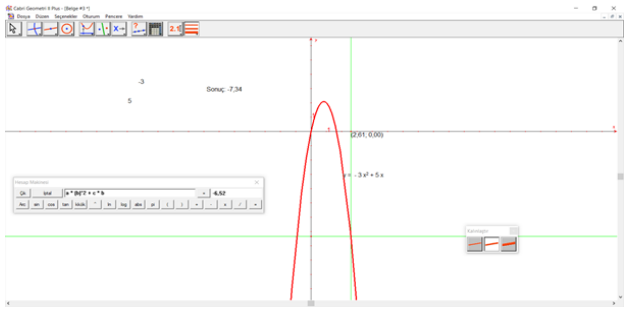


Figure 8. P3 analyzing a parabola on Cabri-Geometry screen

In the paper-and-pencil works conducted by P5 and P6, various strategies were developed for solving and graphing functions such as $f(x)=3x^2$, $f(x)=2x^2+4$, $f(x)= 4x^2-3$, and $f(x =-3x^2+5x$, including understanding the parabola's definition, analyzing coefficient effects on the graph, assigning values, calculating the discriminant, and finding the axis of symmetry. For example, as seen in Figure 9, while working on $f(x)=3x^2$, P5 stated that the parabola approaches the y-axis as the coefficient increases, and noted that the constant term shifts the graph upward along the y-axis in the function $f(x)=2x^2+4$.

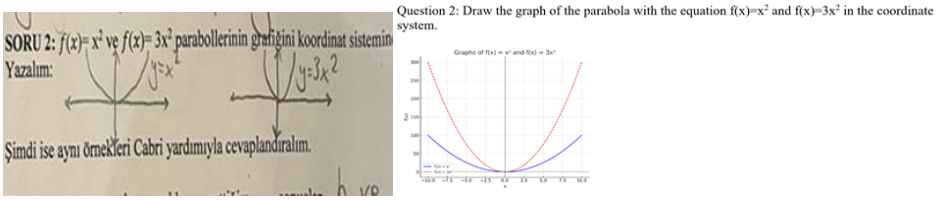


Figure 9. Excerpt from P5's paper showing parabolic solution steps and graph

The theorem-in-action employed by P5 and P6 during this process was identified as: "If the equation of a parabola is to be solved and its graphical representation constructed in the analytic plane, the effects of the equation's coefficients on the graph are analyzed to create the graph." This theorem-in-action is based on the application of 'analytic solving,' 'graph drawing,' and 'geometric effects of coefficients' methods (e.g., point determination, discriminant calculation, axis of symmetry identification, and analysis of the parabola arms' width or direction), relating to the conceptual elements of the action. Reviewing the paper-and-pencil expressions of P5 and P6 reveals that the functional invariant of "performing analytic operations on algebraic equations" underpinning their graphing of parabolas transforms into "establishing visual relations by moving points on the graph" in the Cabri-Geometry environment. P6's statement, "*I multiplied 3 by this number, that is x times x, so I processed x² on the calculator. I transferred the result to the y-axis using the measurement transfer tool.*" indicates the beginning of constructing the function's graphical representation using dynamic tools. This process, reflecting students' shift from a

static and procedural approach to a visual and dynamic one, and Cabri-Geometry's role as an instrument, marks this transition. Overall, these findings show that instrumentation and instrumentalization became more visible in the Cabri-Geometry environment than in the paper-and-pencil environment.

3.3. Instrumented techniques, instrumental genesis process, and instrumented action schemes related to third-degree polynomial equations

This worksheet on cubic polynomials included equations of the forms $f(x)=x^3$, ax^3+bx, ax^3+bx^2+c, and ax^3+c, implemented in both paper-pencil and Cabri-Geometry environments. Participants were expected to solve these equations and graph them accordingly. The worksheet aimed to support visual construction of polynomial concepts through the geometric paths formed by moving points in the analytical plane. Participants mostly used similar instrumented techniques while trying to solve the given functions and draw their graphs in both paper-pencil and Cabri-Geometry environments. Similar techniques in the paper-pencil environment include: point determination, graph translation, factoring, differentiation, root finding, local minimum and maximum concepts, and coefficient change. Similar techniques used in the Cabri-Geometry environment were identified as point determination, perpendicular line drawing, coordinate and intersection point determination, measurement transfer and locus determination, graph changes, and coefficient analysis. The third-degree polynomial solution process, carried out by P1 and P2 in the Cabri environment, provides important clues about instrumental genesis. As shown in Figure 10, P1 initially progressed toward exploring and correctly using basic tools, while P2's guidance supported and further developed this process. Throughout the process, the integration of mathematical concepts, graph analyses, and Cabri-Geometry tools made the preservice teachers' approaches to solving polynomial functions and constructing graphs more concrete and visually grounded. Their functional invariant in the paper-and-pencil environment—"calculating roots algebraically and determining their positions in the analytical plane"—was transformed in the Cabri-Geometry environment into "dynamic point determination, analysis of coefficients' graphical effects, and locus determination." This transformation reflects instrumentation, as the software environment shaped participants' actions and observations.

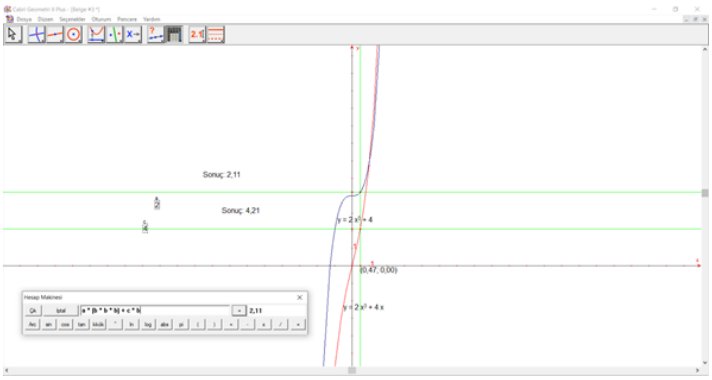


Figure 10. A section from P1's screen recording showing various tool uses related to the cubic polynomials worksheet.

For the function $f(x)=ax^3+bx^2+cx+d$, P1 first assigned initial values of 1 to the coefficients a, b, c, and d and commented on the process as "I think this Cabri-Geometry program should be improved because we assigned coefficient names a, b, and c as text, and when we put numbers next to them, it's strange that it interprets this as $a=1$. It would have been better if we wrote $a=1$ directly in the text." Although this comment is not very software-friendly, it shows that P1 tended to restructure the program according to personal expectations, ignoring Cabri-Geometry's limitations. This situation can be considered an instrumentalization process. P3 and P4 demonstrated complementary use of paper-pencil and Cabri-Geometry environments for cubic polynomials, enabling representation switching and conceptual depth. P4 observed coefficient effects dynamically "*The graph got closer to the y-axis*", while P3 noted visual

changes "*The curvature is changing*". Their theorem-in-action involved: 'For cubic polynomials, use x-value assignment, factoring (e.g., "*I take the parentheses of* x^2"), and differentiation.' Cabri-Geometry facilitated comprehension, as P4 noted: "I understood it when I drew it in Cabri." P3's transition "*I cannot draw this polynomial, but I can in Cabri*" marked Cabri's instrumentalization (Figure 11), revealing distinct instrumental genesis patterns.

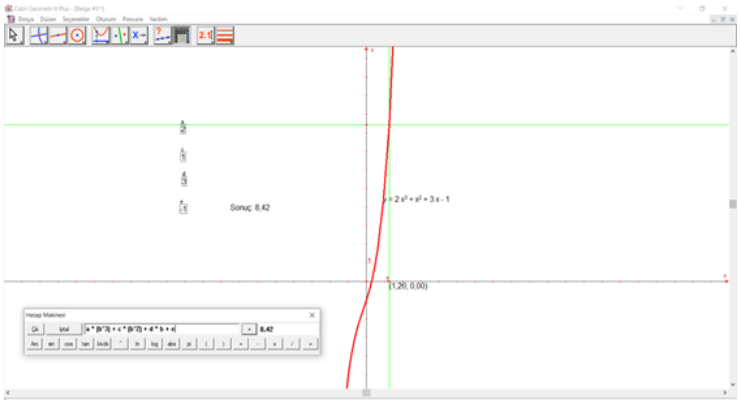


Figure 11. A section from P3's screen recording related to the cubic polynomials worksheet.

The third group of preservice teachers (P5, P6) followed the theorem-in-action 'if the equation of a cubic polynomial is to be solved and graphed analytically, then methods such as differentiation, value assignment, and root finding should be used' while solving the worksheet questions in the paper-pencil environment. Particularly, while solving the function $f(x)=2x^3+4x$, they used the concepts of "differentiation" and "value assignment" together and tried to reach the local maximum point in graphical analysis. When P5 and P6 moved to the Cabri-Geometry environment, they started the process with steps such as axis determination, point placement, and coordinate definition; then conducted graph drawing processes using the functional invariants 'dynamic point determination, analysis of coefficients' graphical effects, and locus determination' by employing Cabri-Geometry tools. P5 observed changes in thickness and orientation of the graph with coefficient changes, and P6 analyzed graph behaviors through variable parameters. These findings suggest that instrumentation and instrumentalization became more visible in the Cabri-Geometry environment than in the paper-pencil environment.

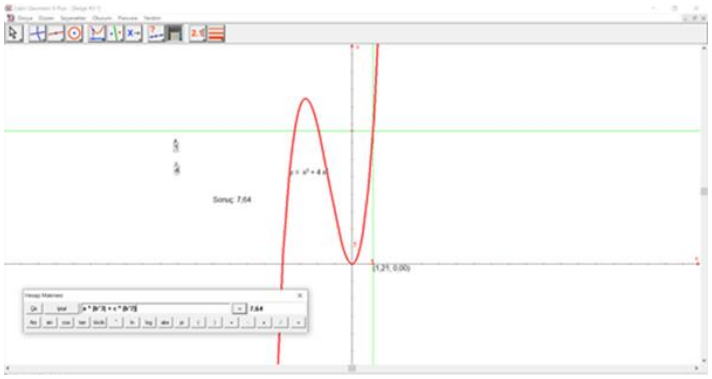


Figure 12. A section from P6's screen recording related to the cubic polynomials worksheet.

Figure 12 shows an example from this dynamic environment process. This study identified instrumented action schemes based on instrumented techniques, operational invariants, theorems-in-action, and concepts-in-action, focusing here solely on the synthesis of operational invariants since the techniques were presented separately. It was found that the operational invariants related to solving and graphing functions were similar across all worksheets in both paper-and-pencil and Cabri-Geometry environments; however, significant differences existed between the two in terms of theorems-in-action, concepts, and operational invariants. These distinctions arose from the differing natures, tools, and representational forms of each environment. While pre-service teachers' actions in the paper-and-pencil setting were mainly rooted in algebraic rules, definitions, and procedural operations, the operational

invariants in Cabri-Geometry were shaped by interactive and visual representations provided by dynamic tools. Thus, the instrumented action schemes in the paper-and-pencil environment reflected algebraic and definitional understanding, whereas those in Cabri-Geometry emphasized visual and functional meaning-making supported by technological interactivity. Table 1 presents the components of the participants' instrumented action schemes for the linear equations worksheet in both environments.

Table 1.

Components of Pre-Service Teachers' Instrumented Action Schemes Related to the Linear Equations Worksheet

Environment	Pre-service Teacher	Operational Invariants	Theorems-in-Action	Concepts-in-Action	Category
Paper-and-Pencil	P1, P2	To draw the graph of a function, a specific point is selected on the coordinate plane, and a line is drawn parallel to the x- or y-axis.	If a graph of a function is to be drawn, a specific point on the coordinate plane should be selected, and a line should be drawn parallel to the x- or y-axis.	Coordinate plane, parallel line, assigning values	Use of coordinate plane, drawing constant function lines
	P3, P4	Two points are determined by assigning values to the x and y variables, and a line is drawn by connecting these points.	If a function's graph is to be drawn, two points are determined by assigning values to x and y, and a line is drawn by connecting these points.	Coordinate plane, assigning values, determining geometric loci, point selection, point connection	Use of coordinate plane, line drawing
	P5, P6	Appropriate points are selected, and a line is drawn by connecting these points.	If a function's graph is to be drawn, appropriate points are determined and connected to draw the line.	Coordinate plane, assigning values, point selection, point connection	Use of coordinate plane, slope analysis, line drawing
Cabri-Geometry	P1, P2	The graph of constant and linear functions emerges as the initial point moves in a specific direction and ratio. The starting and locus points determine the graph.	If the graph of a constant or linear function is to be drawn, the movement of the initial point in a specific direction and rate generates the graph. If the slope and the y-intercept of a line are known, its position can be determined. If a point moves in a certain direction, the trace it leaves defines the graph of the function.	Initial point, locus point, y-intercept, slope of the line	Creating point traces, constructing constant function graphs, observing line movement, slope analysis
	P3, P4	The initial and locus points determine the graph of the constant function.	If the initial point of a constant or linear function moves in a particular direction and at a certain rate, the trace of this movement creates the graph.	Initial point, locus point	Creating point traces, observing line movement, constructing constant function graphs
	P5, P6	The initial and locus points determine the graph of the constant function.	If the slope of a line and its y-intercept are known, these two pieces of information determine the position of the line.	Y-intercept, slope of the line, trace generation	Slope analysis, constructing constant function graphs, creating point traces

An examination of Table 1 reveals that the functional invariants, applied theorems, and applied concepts that constitute pre-service teachers' instrumental action schemes show differences between the paper-and-pencil environment and the Cabri-Geometry environment. In the paper-and-pencil environment,

participants constructed graphs of constant and linear functions through static reasoning processes such as assigning values, selecting points, analyzing slopes, and using the coordinate plane. In contrast, the Cabri-Geometry environment provided opportunities for dynamic exploration: preservice teachers tracked the movement of points, observed how graphs emerged from the movement of the starting point, and interactively analyzed the slope and position of lines. Cabri-Geometry's dynamic features enhanced visualization, allowing participants to relate slope and constant terms to graphical changes and interpret the graph through movement and path tracing. Overall, while tasks performed with paper and pencil supported procedural understanding through static representations, the Cabri-Geometry environment supported more dynamic and interactive forms of meaning-making.

Table 2.

Components of Pre-Service Teachers' Instrumented Action Schemes Related to the Quadratic EquationsWorksheet

Environment	Pre-service Teacher	Operational Invariants	Theorems-in-Action	Concepts-in-Action	Category
Paper-and-Pencil	P1, P2	To draw the graph of a function, a specific point is selected on the coordinate plane, and a line is drawn parallel to the x- or y-axis.	If a graph of a function is to be drawn, a specific point on the coordinate plane should be selected, and a line should be drawn parallel to the x- or y-axis.	Coordinate plane, parallel line, assigning values	Use of coordinate plane, drawing constant function lines
	P3, P4	Two points are determined by assigning values to the x and y variables, and a line is drawn by connecting these points.	If a function's graph is to be drawn, two points are determined by assigning values to x and y, and a line is drawn by connecting these points.	Coordinate plane, assigning values, determining geometric loci, point selection, point connection	Use of coordinate plane, line drawing
	P5, P6	Appropriate points are selected, and a line is drawn by connecting these points.	If a function's graph is to be drawn, appropriate points are determined and connected to draw the line.	Coordinate plane, assigning values, point selection, point connection	Use of coordinate plane, slope analysis, line drawing
Cabri-Geometry	P1, P2	The graph of constant and linear functions emerges as the initial point moves in a specific direction and ratio. The starting and locus points determine the graph.	If the graph of a constant or linear function is to be drawn, the movement of the initial point in a specific direction and rate generates the graph. If the slope and the y-intercept of a line are known, its position can be determined.	Initial point, locus point, y-intercept, slope of the line	Creating point traces, constructing constant function graphs, observing line movement, slope analysis
	P3, P4	The initial and locus points determine the graph of the constant function.	If the initial point of a constant or linear function moves in a particular direction and at a certain rate, the trace of this movement creates the graph.	Initial point, locus point	Creating point traces, observing line movement, constructing constant function graphs

	P5, P6	The initial and locus points determine the graph of the constant function.	If the slope of a line and its y-intercept are known, these two pieces of information determine the position of the line.	Y-intercept, slope of the line, trace generation	Slope analysis, constructing constant function graphs, creating point traces

Table 2 presents in detail the operational invariants, theorems-in-action, concepts-in-action, and categories that emerged during pre-service teachers' instrumental genesis processes while working on quadratic equations worksheets in both paper-and-pencil and Cabri-Geometry environments. In the paper-and-pencil environment, the participants focused on analyzing the effects of coefficients, constructing graphs, and determining the axis of symmetry, vertex, roots, and critical points through algebraic methods such as differentiation, value assignment, and factorization. In contrast, in the Cabri-Geometry environment, they employed dynamic tools to generate and trace points, observe the effects of coefficients, and construct graphs interactively, which facilitated relational understanding between coefficients, roots, and graphical behavior. Overall, the findings indicate that while paper-and-pencil tasks supported analytical and procedural understanding through algebraic operations, the Cabri-Geometry environment enabled visual and dynamic exploration that strengthened the integration of conceptual understanding with technological tools. Table 3 presents the components of pre-service teachers' nstrumented action schemes related to the cubic polynomials worksheet in both environments.

Table 3.

Components of Pre-Service Teachers' Instrumented Action Schemes Related to the Cubic Polynomials Worksheet

Environment	Pre-service Teacher	Operational Invariants	Theorems-in-Action	Concepts-in-Action	Category
Paper-and-Pencil	P1, P2	To construct the graph of a cubic polynomial, values are assigned to the x-variable to calculate points; roots are found via factoring; a sign table is created to determine the sign of each interval; roots and critical points are marked on the coordinate system; and the effect of coefficients is analyzed to construct the graph.	If a cubic polynomial is to be solved and graphed, its roots must be determined and accurately placed on the coordinate plane. If such a polynomial is to be analyzed, values should be assigned to x to determine points, and factoring should be used to find the roots.	Application of methods such as factoring, analyzing the sum and product of roots, or graphical methods (e.g., determining roots through factoring, computing sum/product based on coefficients, conducting visual graph analysis). Also includes point plotting via substitution, factoring, and creating sign charts.	Use of coordinate system, identification of roots and critical points
	P3, P4	To construct the graph, values are assigned to the x-variable; roots are found through factoring; and critical points are identified using derivative methods. Roots are marked on the coordinate plane and the graph is drawn accordingly.	If a cubic polynomial is to be solved and graphed in the analytic plane, values are assigned to x, and methods such as factoring and differentiation are used to find the roots and critical points.	Applying value substitution, factoring, or derivative methods (e.g., finding roots via factoring, plotting points, performing visual analysis on the graph)	Use of coordinate system, identification of roots and critical points, differentiation, graph construction

	P5, P6	Roots and critical points are identified through differentiation, value substitution, and root-finding methods. These are marked on the coordinate system, and the graph is constructed accordingly.	If a cubic polynomial is to be analyzed in the analytic plane, differentiation, substitution, and root-finding techniques should be applied.	Implementation of derivative, substitution, and root-finding methods (e.g., locating roots via derivatives, point plotting, analyzing function behavior)	Use of coordinate system, identification of roots and critical points, differentiation, graph construction
Cabri-Geometry	P1, P2	For solving and graphing cubic polynomials, dynamic tools are used to identify roots, find x-intercepts, and analyze local maxima, minima, and slope behavior within the Cabri-Geometry environment.	If the real roots of a polynomial are to be determined, they correspond to the x-intercepts of the function's graph. A cubic polynomial graph includes at least one local maximum and one local minimum; at these points, the slope is zero, and the function's behavior changes. If the polynomial is factorable, its roots can be directly found by setting each factor equal to zero. According to the Intermediate Value Theorem, if a function is continuous and has opposite signs at the endpoints of an interval, it must have at least one real root within that interval.	Roots, x-intercepts, factoring, perfect square expansion, local maxima and minima, slope variation	Identification of roots and critical points, graph construction, analysis of function behavior through coefficients
	P3, P4	In the Cabri-Geometry environment, dynamic point manipulation, graphical analysis of coefficient effects, and construction of geometric loci are used. Additionally, the roots, local maxima and minima, and x-intercepts of the function are identified.	If a cubic polynomial has real roots, they appear as x-intercepts of its graph. If its coefficients change, these changes affect the width, direction, and the position of local extrema.	Dynamic point manipulation, graphical effect of coefficients, construction of geometric loci, identification of local extrema and roots, determination of x-intercepts	Identification of roots and critical points, graph construction

P5, P6	Dynamic point manipulation, analysis of coefficient effects, and creation of geometric loci are performed using Cabri tools. Additionally, the roots, local extrema, and x-intercepts are identified.	If a cubic polynomial has real roots, they correspond to the x-intercepts on the graphical representation. If the derivative of the function is taken and set to zero, the resulting x-values give the local maximum and minimum points, indicating where the behavior of the function changes.	Dynamic point manipulation, root-finding, differentiation, coefficient analysis, geometric locus creation, identification of local extrema	Identification of roots and critical points, graph construction, analysis of function behavior through coefficients

Table 3 presents in detail the operational invariants, theorems-in-action, concepts-in-action, and categories that emerged from pre-service teachers' instrumental genesis processes as they engaged with the cubic polynomial worksheet in both paper-and-pencil and Cabri-Geometry environments. In the paper-and-pencil environment, participants primarily focused on categories such as the use of coordinate systems, identification of roots and critical points, taking derivatives, and graph construction. In contrast, in the Cabri-Geometry environment, participants were observed to engage in processes such as identifying roots and critical points, analyzing function behavior through coefficient changes, and graph construction. Based on these findings, common categories identified across both environments were clustered into broader themes.

Themes in the paper-and-pencil environment

In the paper-and-pencil environment, the theme Transformation of Conceptual Knowledge into Procedures highlighted how conceptual understandings—such as injectivity and surjectivity—were transformed into graphical and functional representations. The discussions around the injectivity and surjectivity of linear functions significantly contributed to the emergence of this theme, while debates on the one-to-one nature of quadratic functions and the analysis of the domain and symmetry behavior of polynomial functions further enriched this category. Another theme, Procedure-Oriented Representations and Algebraic Transformations, reflected preservice teachers' preferences for employing algebraic strategies—such as sign charts, root finding, and critical point determination—to graph polynomial functions in the third worksheet. Techniques like substitution, equation manipulation, and completing the square further illustrated the algebraic lens through which participants navigated multiple representations. The theme Difficulties in Representation and Conceptual Limitations arose from the challenges preservice teachers experienced during graph construction. Common issues included the inability to accurately define domains and ranges, the misinterpretation of symmetry axes, and the tendency to treat graphs solely as outputs of algebraic computations. These problems pointed to conceptual limitations in participants' understanding, while difficulties in correctly depicting directional changes and slope behavior in cubic functions further underscored persistent gaps in conceptual representation.

Themes in the cabri-geometry environment

In the Cabri-Geometry environment, the theme Conceptual Understanding through Dynamic Representation emerged, reflecting how preservice teachers utilized dynamic tools to analyze the structural properties of functions via their graphical representations. Transformations such as reflections, the identification of symmetry axes, and the linking of graphical and algebraic properties revealed that

participants did not rely solely on algebraic thinking but engaged deeply with visual and structural reasoning. For instance, in the first worksheet, P1 and P2 used dynamic trace tools to represent constant functions; in the second, P3 and P4 analyzed symmetry axes through dynamic manipulation; and in the third, P5 and P6 examined local maxima and minima in cubic functions—all of which exemplified this theme.

Another prominent theme was Relating Mathematical Concepts through Visualization, which highlighted how the Cabri-Geometry environment enabled preservice teachers to establish connections between mathematical concepts through visual tools. During the construction of linear and constant function graphs, participants identified the relationship between constant terms and slope through the traces of moving points. Similarly, in quadratic functions, they connected the vertex, symmetry axis, and coefficient relationships within a unified graphical framework. In cubic graphs, the visualization of slope variation and its relationship with roots and local extrema further illustrated this theme, showing how real-time graphical manipulation fostered an integrated understanding of algebraic expressions and their visual counterparts. A further theme, Dynamic Analysis of Function Behavior in an Interactive Environment, captured how participants examined temporal changes and interactive visuals to analyze functions dynamically. Through the use of traces, slope variations, and directional shifts, they explored the extrema of cubic functions as well as the width and direction changes of parabolas tied to coefficients. Across worksheets, continuous graph modifications revealed functions as dynamic rather than static structures, while Cabri's tools—such as intersection points, rays, and movable points—facilitated this parametric exploration.

4. RESULTS AND DISCUSSION

In line with the aim of this study, the findings indicated that technological learning environments supported preservice teachers' conceptual understanding, particularly in relation to graph construction and the visualization of functions and their algebraic and graphical representations. In this section, the results are presented in accordance with the themes that emerged during the data analysis process and are discussed in light of the relevant literature and conceptual framework. The similarities and differences between the present findings and previous studies are discussed with justifications, the strengths and limitations of the research are outlined, and its contribution to the field is emphasized. In addition, implications for practice are provided along with suggestions for improving the quality of teaching and for future research. This interpretation is also in line with recent reviews emphasizing that digital technologies increasingly reshape mathematical meaning-making, representation, and visualization in mathematics education.

When the instrumental action schemes of preservice teachers in the paper-and-pencil environment were examined, it was observed that they generally structured functions and their algebraic and graphical representations within a fixed sequence of procedural steps. Their solution processes were shaped by an approach focusing on the order of operations and the preservation of algebraic relations. The instrumental action schemes constructed in this environment were identified as follows: "If linear, quadratic, and polynomial functions are to be solved and graphed in the paper-and-pencil environment, necessary points are determined by assigning values to the variables x and y, and a graph is formed by connecting these points. When required, factoring and derivative methods are used to find the roots of the equation, the vertex, or the minimum and maximum points of the graph; these values are then placed on the coordinate plane to construct the graph." The results showed that preservice teachers predominantly adopted a procedural, rule-based approach and preferred fixed solution paths, while some also demonstrated efforts toward conceptual understanding and developed alternative strategies. Although most participants tended to transform these representations procedurally rather than question the structural meaning of variables, individual differences revealed the diversity of learning processes. These results are largely consistent with the literature. Drijvers (2019) stated that procedural knowledge may delay conceptual understanding; Kieran and Drijvers (2006) emphasized the dominance of procedural thinking and the neglect of structural comprehension; and Trouche (2005) highlighted that

conceptual understanding remains limited when instrumental genesis does not occur. The preservice teachers' reliance on hand-drawn axes and the placement of points on the coordinate plane indicated that graphs were often limited to formal accuracy. Deficiencies were observed particularly in recognizing symmetry, identifying special points such as vertices, and understanding domains and ranges. This finding suggests the need for approaches to graphing in algebra education that emphasize not only procedural but also conceptual dimensions. However, Xu (2025) presented a different perspective, emphasizing that the paper-and-pencil environment is not merely a limited structure supporting procedural thought, but also plays a significant role in the development of fundamental skills and can adapt to different learning styles. Thus, the present findings differ from those of Xu. Kaplan (2010) found that students and preservice teachers mostly focused on algebraic operations in graph construction while neglecting functional and geometric properties. Similarly, Gürbüz and Şahin (2015) drew attention to the difficulties learners experience in transitions between multiple representations, while Eroğlu (2020) showed that preservice mathematics teachers may experience errors and limitations in representation transformations. In the present study, comparable difficulties became visible in the paper-and-pencil environment, where graph construction was often reduced to procedural steps rather than supported by relational meaning-making. This suggests that while preservice teachers were successful in positioning graphs correctly on the coordinate plane, they struggled to explain behavioral properties such as monotonicity and symmetry. In contrast, Bautista et al. (2015) indicated that experienced teachers, through their professional knowledge and classroom practice, were able to interpret and convey these properties more effectively. From a broader perspective, this result also supports recent discussions that digital technologies do not simply replace paper-and-pencil work, but rather reveal different forms of mathematical activity and different opportunities for conceptual development.

When the instrumental action schemes constructed in the dynamic geometry environment were examined, it was observed that preservice teachers' processes of solving functions and constructing their graphs were structured through the use of tools provided by the software. Specifically: "If linear, quadratic, and polynomial functions are to be solved and graphed in the dynamic geometry environment, coefficients and constants are first defined using the numeric tool provided by the environment, then these values are processed through the calculator tool to obtain the result (y). This result is transferred to the y-axis using the measurement transfer tool; perpendiculars are drawn from the initial and final points to determine their intersection. This intersection point is referred to as a locus or trace point. By employing the locus tool, the trace generated relative to the initial point x produces the graph of the function." The results indicated that preservice teachers adopted meaning-oriented and relational approaches in this environment, attempting to build conceptual understanding by observing relationships between variables. They not only performed operations but also considered the mathematical structures represented by those operations and questioned what the symbols (coefficients and variables) represented. These findings demonstrate that preservice teachers constructed mathematical structures in the dynamic geometry environment on the basis of conceptual understanding, consistent with Haspekian et al. (2024), Bråting and Kilhamn (2021), and Drijvers (2015), who emphasized that dynamic environments strengthen conceptual links between the algebraic and graphical representations of functions. The study also revealed that preservice teachers regarded technology not only as a procedural tool but as a medium that structures thinking and supports the development of instrumental action schemes, a finding related to the theme "Conceptual Understanding through Dynamic Representation." In this respect, the present findings extend the literature by showing that dynamic geometry environments support preservice teachers not only in moving between representations, but also in constructing richer conceptual relations among those representations. However, this contribution should be interpreted cautiously. Chen (2025) reported that although dynamic graphical representations may enhance certain forms of reasoning, some learners still experience difficulties in coordinating graphical and algebraic meanings. In this respect, the present findings suggest

that the conceptual advantages of dynamic geometry environments are not automatic, but depend on how learners engage with the available representations

Nevertheless, different findings exist in the literature. Baccaglini-Frank and Antonini (2016) argued that in dynamic environments students may focus on specific patterns or context-dependent observations, which may hinder generalizable conceptual understanding. However, one of the participants (P1) expressed the opposite view, emphasizing Cabri-Geometry as a strong tool for meaning-making and graphical interpretation, stating: "Now, regardless of the function given, for example the closed form of the circle $x^2+y^2=1$, I can rearrange it in terms of y, draw it in Cabri-Geometry, and interpret the graph thanks to this representation." In graph construction within the dynamic geometry environment, preservice teachers defined the coordinate axes and then used measurement transfer, perpendicular lines, and locus tools to construct graphs. For instance, when graphing a cubic polynomial, they created a locus point, manipulated it to observe dynamic changes, and identified roots and local extrema. Concepts such as "locus point," "symmetry axis," and "dynamic variation" became prominent in their action schemes. These findings indicate that graph construction moved from being a purely technical activity to becoming a process of conceptual learning. By observing parameter-dependent transformations, participants made intuitive inferences regarding slope, direction, and critical points. This finding corresponds with the theme "Relating Mathematical Concepts through Visualization" and is consistent with Mariotti (2000), Zhang et al. (2025), and Weigand et al. (2024). At the same time, Bach and Bickner-Achsbachs (2022) demonstrated that digital tools do not always lead students to mathematically productive communication, and that tool use can sometimes remain mechanical or trial-and-error based. This perspective is important for interpreting the findings, as although Cabri-Geometry facilitated concept development in many cases, some participants also demonstrated limited or incomplete conceptualization when using the software. At the same time, some studies emphasized that the effects of dynamic environments are conditional. Donnelly-Hermosillo et al. (2020) noted that graph technologies enhance inquiry-based reasoning in the short term but do not consistently affect the long-term retention of conceptual knowledge. Similarly, Pittalis and Drijvers (2023) observed that dragging-based activities often improve procedural competence but make limited contributions to conceptual understanding. Thus, while dynamic software facilitates graph construction, it does not always directly lead to conceptual understanding; therefore, such environments need to be supported with teacher-guided, structured activities.

The results of this study revealed significant differences between preservice teachers' instrumental action schemes in paper-and-pencil and dynamic geometry environments. In the paper-and-pencil environment, solution and graphing processes were largely reduced to procedural steps and formal accuracy, with critical features such as roots, vertices, and symmetry axes often overlooked. In contrast, in the dynamic environment preservice teachers adopted a more conceptual and relational approach, directly observing features such as monotonicity, turning points, and symmetry through locus tools. The observation of parameter changes in real time allowed them to establish meaningful connections between the algebraic and graphical representations of functions, while instant feedback mechanisms supported error detection and cognitive restructuring. However, recent research also suggests that technology-based environments can pose different types of challenges depending on the nature of the task. For example, Kirsten, Griefrath, and Emrich (2026) found that tasks involving dynamic geometry software can be perceived as more challenging in some contexts than tasks performed with paper and pencil. Therefore, the differences found in this study should not be interpreted as indicating that dynamic environments are always easier, but rather as evidence that they provide different cognitive and representational capabilities. These findings are consistent with Bokhove and Drijvers (2012), who emphasized the cognitive role of digital tools; Wei et al. (2023), who highlighted technology as a bridge to conceptual understanding; and Yerushalmy (2006) and Mariotti (2000), who pointed to the value of dynamic environments in analyzing graphical behavior. However, as Barabash (2019) cautioned, overly restrictive structures of digital tools may limit creativity, a limitation also observed in this study, where the software's rigid formats restricted some participants' ability to develop alternative solution strategies.

Conclusion

This study examined preservice teachers' processes of constructing functions and their algebraic and graphical representations in paper-and-pencil and dynamic geometry environments within the framework of instrumental genesis theory. The findings demonstrated that the environment directly influenced preservice teachers' instrumental action schemes. In the paper-and-pencil environment, they generally adopted procedural, fixed, and accuracy-oriented strategies, whereas in the dynamic environment they engaged in interactive and conceptually oriented approaches. The instrumented action schemes developed in the dynamic environment were characterized by the ability to transition between multiple representations, observe and interpret graphical behaviors, and explain relationships between variables. This study clearly demonstrates that preservice teachers' instrumented action schemes are shaped not only by the digital or traditional tools they use, but also by their interaction with those tools. Understanding the dynamics underlying this interaction positions instrumental genesis theory as an indispensable theoretical key for interpreting the nature and transformation of learning.

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APPENDİX

Appendix-1. Data Collection Tools, Purpose, and Analysis Process Based on Research Problems

Research Sub-Problems	Analysis Process	Data Collection Tools
1. What are the pre-service teachers' instrumented action schemes regarding algebraic expressions in the paper-pencil environment?	The worksheet (paper-pencil environment questions), audio recordings, and written interviews were analyzed to determine the instrumented techniques used, the instrumental genesis process, and the instrumented action schemes in the paper-pencil environment; based on these schemes, codes and themes were created.	•Worksheets (Paper-pencil questions) • Audio Recording • Written and face-to-face Interviews
2. What are the pre-service teachers' instrumented action schemes regarding algebraic expressions in the dynamic geometry environment?	The worksheet (Cabri-Geometry environment questions), audio and screen recordings, and written and oral interviews were analyzed to determine the instrumented techniques used, the instrumental genesis process, and the instrumented action schemes in the dynamic geometry environment; based on these schemes, codes and themes were created.	• Worksheets (Cabri Geometry questions) • Audio Recording • Screen Recording • Written and face-to-face Interviews
3. What are the differences between the pre-service teachers' instrumented action schemes regarding algebraic expressions in the paper-pencil environment and the dynamic geometry environment?	Data from both environments (worksheets, audio recordings, screen recordings, written and oral interviews) were analyzed. The identified instrumented action schemes were compared, and based on this comparison, codes and themes were formed.	• Worksheets • Audio Recording • Screen Recording • Written and face-to-face Interviews

UZUN ÖZET

1. GİRİŞ

Matematik eğitimine teknolojinin entegrasyonu, özellikle Cabri-Geometri, Geometer's Sketchpad ve GeoGebra gibi dinamik geometri yazılımları aracılığıyla, soyut cebirsel kavramların öğretiminde dönüştürücü bir potansiyel sunmaktadır. Bu yazılımlar, fonksiyonların cebirsel, grafiksel ve sayısal olmak üzere çoklu temsiller arasında dinamik geçişlere olanak sağlayarak öğrenme sürecini zenginleştirmektedir (Zbiek & Hollebrands, 2008). Geleneksel kâğıt-kalem ortamları kavramsal anlamlandırma açısından sınırlayıcı olsa da teknolojik araçlar değişken ve eşitlik gibi temel fakat öğrenciler için güçlük oluşturan fonksiyonların somutlaştırılmasına ve temsiller arası bağlantıların güçlendirilmesine katkı sunmaktadır (İlhan & Aslaner, 2020; Trouche, 2005). Bu tür teknolojik entegrasyonun etkililiği, öğretmenlerin matematiksel bilgilerini gelişen teknolojik yeterliklerle bütünleştirebilme becerilerine kritik ölçüde bağlıdır (Mishra & Koehler, 2006). Bu bütünleşme süreci, araçların bilişsel bir enstrümana dönüşümünü açıklayan ve eyleme dayalı enstrümanlı şemaların gelişimini merkeze alan enstrümantal oluşum kuramı çerçevesinde ele alınmaktadır (Verillon & Rabardel, 1995; Trouche, 2004). Matematik didaktiğinde bu yaklaşımın önemi artmasına rağmen (Trouche, 2020), öğretmen adaylarının dinamik ortamlarda temsiller arası geçişlerde geliştirdikleri şemaların anlaşılmasına yönelik çalışmalar sınırlı kalmaktadır (Turgut & Drijvers, 2021) ve mevcut araştırmaların çoğu öğretmen eğitimi yerine öğrenci öğrenimine odaklanmaktadır (Çetin, 2017). Bu çalışma, enstrümantal oluşum kuramını (Verillon & Rabardel, 1995) temel alarak öğretmen adaylarının kâğıt-kalem ve Cabri-Geometri ortamlarında geliştirdikleri enstrümanlı eylem şemalarını karşılaştırmakta; ayrıca adayların alan bilgisi ile teknolojik alan bilgilerini nasıl bütünleştirdiklerini inceleyerek literatüre özgün bir katkı sunmaktadır (Drijvers & Trouche, 2008).

2. YÖNTEM

Bu çalışma, öğretmen adaylarının bilgisayar destekli ve kâğıt-kalem temelli öğrenme ortamlarında geliştirdikleri enstrümantal süreçleri incelemek amacıyla nitel araştırma desenine dayalı bir öğretim deneyi (teaching experiment) olarak yapılandırılmıştır (Steffe & Ulrich, 2014; Yenmez, 2022). Öğretim deneyi yaklaşımı, araştırmacının Cabri-Geometry destekli öğrenme etkinliklerini tasarlamasına, öğretmen adaylarının düşünme süreçleri hakkında varsayımlar üretmesine ve bu varsayımları süreç içinde elde edilen veriler ışığında sürekli olarak test edip yeniden düzenlemesine imkân tanımıştır. Araştırma, Bursa Uludağ Üniversitesi'nde öğrenim gören ikinci sınıf matematik öğretmenliği programı öğrencileri ile yürütülmüştür. Çalışmanın sınıf uygulamaları, toplam 34 öğretmen adayı ile gerçekleştirilmiş; ancak, enstrümantal oluşum süreçlerinin daha derinlemesine analiz edilebilmesi amacıyla, katılımcılar arasından farklı akademik başarı düzeylerini temsil eden altı öğretmen adayı, amaçlı örnekleme yöntemiyle seçilmiştir. Ayrıntılı nitel analiz bu altı katılımcı üzerinde yürütülmüş; böylece bireysel enstrümanlı eylem şemalarının gelişimi incelenmiştir. Veriler; çalışma kâğıtları, yüz yüze ve yazılı görüşmeler, ses kayıtları ve bilgisayar ekran kayıtları aracılığıyla toplanmıştır. Doğrusal, prabolik ve polinom fonksiyonlara yönelik üç yapılandırılmış çalışma kâğıdı, öğretmen adaylarının Cabri-Geometry ve kâğıt-kalem ortamlarında geliştirdikleri enstrümanlı eylem şemalarının karşılaştırılmasına olanak sağlamıştır. Ekran ve ses kayıtları, katılımcıların teknolojik araçlarla etkileşimlerini ve düşünme süreçlerini eşzamanlı olarak izleme fırsatı sunarken; görüşmeler, kavramsal anlamalar, zorlanılan noktalar ve enstrümantal oluşum süreçlerinin gelişimini ortaya çıkarmıştır. Verilerin analizinde, enstrümantal oluşum teorisi çerçevesinde içerik analizi kullanılmıştır. Bu kapsamda, veriler önce çözümlenmiş; ardından kodlar, kategoriler ve temalar oluşturularak öğretmen adaylarının enstrümanlı eylem şemaları değerlendirilmiştir. Araştırmanın geçerlik ve güvenirliği; uzman görüşü, veri çeşitliliği, katılımcı ifadelerinin doğrudan alıntılarla desteklenmesi, ayrıntılı betimleme ve uzun süreli etkileşim yoluyla sağlanmıştır. Tüm süreç boyunca katılımcı mahremiyeti korunmuş ve araştırma, etik kurul tarafından onaylanan izin doğrultusunda yürütülmüştür.

3. BULGULAR, TARTIŞMA VE SONUÇLAR

Kâğıt-kalem ortamında öğretmen adaylarının enstrümantal eylem şemaları, değer verme, nokta yerleştirme, çarpanlara ayırma ve türev temelli kök ya da tepe noktası belirleme gibi sabit işlemsel adımlara dayalı olarak yapılandırılmış ve grafiklerin biçimsel doğruluğa odaklandığı görülmüştür. İşlemsel yaklaşımın baskın olması; tanım kümesi ve değer kümesini belirleme, simetriyi yorumlama ve fonksiyonel davranışı anlama gibi alanlarda güçlükler doğurmuş, bu da kavramsal sınırlılıkları ortaya koymuştur. Bu durum, işlemsel düşünmenin sınırlarına dikkat çeken çalışmalarla uyumludur (Drijvers, 2019; Kieran & Drijvers, 2006; Trouche, 2005). Buna karşılık Cabri-Geometry ortamında öğretmen adayları, dinamik temsile dayalı anlam yönelimli enstrümantal eylem şemaları geliştirmiştir. Ortamın sunduğu dinamik araçlar, hesaplayıcı, ölçü aktarımı, dikmeler ve iz (locus) araçları kullanılarak grafikler oluşturulmuş; bu süreç, yapısal akıl yürütmeyi ve görsel-kavramsal anlayışı desteklemiştir. Dinamik etkileşim, fonksiyonların cebirsel ve grafiksel temsiller arasındaki ilişkilerin kurulmasını kolaylaştırmış ve bu bulgu, teknolojik araçların kavramsal katkılarını vurgulayan çalışmalarla paralellik göstermektedir (Haspekian vd., 2024; Bråting & Kilhamn, 2021; Drijvers, 2015). Bununla birlikte, literatürde dinamik ortamların etkisinin koşula bağlı veya sınırlı olabileceğini belirten çalışmalar da mevcuttur (Baccaglini-Frank & Antonini, 2016; Pittalis & Drijvers, 2023). Genel olarak sonuçlar, iki ortam arasında belirgin farklılıklar ortaya koymuştur: kâğıt-kalem temelli süreçler çoğunlukla işlemsel ve sınırlı kalırken, dinamik geometri ortamı ilişkisel akıl yürütmeyi, anlık keşfi ve bütünleşik kavramsal anlayışı güçlendirmiştir. Bununla birlikte Barabash'ın (2019) da belirttiği gibi, araştırmada yazılımın katı yapısal kısıtlarının zaman zaman öğretmen adaylarının yaratıcılığını sınırladığı ve alternatif çözüm stratejileri geliştirme kapasitelerini daralttığı gözlenmiş; bu durum teknolojik araçların öğrenme süreçlerini hem destekleyip hem de sınırlayabilen çift yönlü bir yapıya sahip olduğunu göstermiştir.

Bu çalışma, öğretmen adaylarının kâğıt-kalem ve dinamik geometri ortamlarında fonksiyonların cebirsel ifadeleri ve grafiklerini oluşturma süreçleri enstrümantal oluşum teorisi çerçevesinde incelemiştir. Bulgular, öğrenme ortamının öğretmen adaylarının enstrümanlı eylem şemalarını doğrudan etkilediğini göstermiştir. Kâğıt-kalem ortamında katılımcılar ağırlıklı olarak işlemsel, sabit ve doğruluk odaklı stratejiler benimserken, dinamik geometri ortamında etkileşimli ve kavramsal odaklı yaklaşımlar sergilemişlerdir. Dinamik ortamda geliştirilen enstrümanlı eylem şemaları, çoklu temsiller arasında geçiş yapabilme, grafiksel davranışları gözlemleyip yorumlayabilme ve değişkenler arasındaki ilişkileri açıklayabilme özellikleri ile tanımlanmıştır. Bu çalışma, öğretmen adaylarının enstrümanlı eylem şemalarının yalnızca kullandıkları teknolojik veya geleneksel ortamların (kâğıt-kalem) araçlarla değil, aynı zamanda bu araçlarla kurdukları etkileşimle de şekillendiğini açıkça ortaya koymaktadır. Bu etkileşimin dinamiklerini anlamak, öğrenmenin doğasını ve dönüşümünü yorumlamak için enstrümantal oluşum teorisini vazgeçilmez bir teorik çerçeve olarak konumlandırmaktadır.

ETHICAL APPROVAL

In this study, all rules stated to be followed within the scope of the "Higher Education Institutions Scientific Research and Publication Ethics Directive" were followed. None of the actions specified under the title of "Actions Violating Scientific Research and Publication Ethics," which is the second part of the directive, have been carried out.

Ethics Committee Approval Information

Ethical committee: Bursa Uludag University

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CONTRIBUTION OF RESEARCHERS

The first author contributed 60% and the second author contributed 40% to this study.

First Author: Contributed to the design of the study, data analysis, and reporting of the findings.

Second Author: Contributed to determining the research methodology, data analysis, providing consultation throughout the research process, and conducting the validity and reliability procedures.

CONFLICT OF INTEREST

The authors declare that they have no financial or personal relationships that could have influenced the work reported in this study. The authors further declare that there are no conflicts of interest.