Comparison of Bezout Elimination Method and a New Method for Solving Nonlinear Systems of Equations of Second Order

Ercan Duzgun[1], ***Osman Kopmaz***[1] **CT**

[1]Mechanical Engineering Department, Bursa Uludag University, Bursa, Turkey

A new mathematical method is investigated for solving nonlinear system of equations with three unknown variables in three equations in a special format. The proposed method is based on finding the roots of a second order algebraic equations (polynomials) which have a symmetry between the unknowns. In this case the roots of any two of these equations are found in terms of one of these unknowns. Afterwards these roots are substituted in the remaining equation, which leads to a 32 degree polynomial after some symbolic equations, while using the Bezout Elimination method, it yields to a 16th degree polynomial by calculating symbolic determinants of 4x4 and 6x6 matrices which is comparatively difficult. The proposed method is easy to implement while still provides all roots. In the practice, the Stewart-Gough platforms used in the robotics/airplane simulators need the solutions of this kind of system of equations. The presented technique can be used for the forward kinematics problem to find possible manipulator assembly modes as well as any kind of nonlinear systems of equations in the same format as given here by using only simple mathematical operations that doesn't need special mathematical techniques.

Keywords: Bezout, nonlinear, polynomials, robotics, roots.

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http://math2022.tubitak.gov.tr

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About

Studies on Scientific Developments in Geometry, Algebra and Applied Mathematics

The aim of this webinar is to create platform where scientists from different fields (algebra, geometry, and applied mathematics) from all over the world can share their great experiences and, at the same time, raise awareness of career opportunities available to postgraduate students/graduate fellows and researchers. Therefore, this event aims to discuss vigorously the hottest and interdisciplinary topics in the related fields of mathematics at advanced and engaging levels.

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Talha Arikan		

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Zekeriya Arvasi		

Schedule

IT: Invited Talk, IS: Invited Speaker, CT: Contributed Talk.
Webinar is going to be going to be accessible at GMT+03:00 (Istanbul time zone) through the following link:
https://tubitak-gov.zoom.us/j/93192091965
Webinar ID: 931 9209 1965

Tuesday, February 01

9.00–9:15	**Welcome**		
9:15–09:55	IT	**Hasan Mandal** President, TUBITAK	Opening Speech
09:55–10:00	**Break**		
Moderator	**Figen T. Mutlu (figent@eskisehir.edu.tr)**		
10:00–10:55	IS	**Vesselin S. Drensky** Sofia, Bulgaria	Non-finitely Based and Limit Varieties of Algebraic Systems
10:55–11:00	**Break**		
Moderator	**Hande Uslu (usluh@yildiz.edu.tr)**		
11:00–11:20	CT	**Ismail Aslan** Ankara, Turkey	Φ-variational Approximation by Mellin-type Nonlinear Integral Operators
11:20–11:40	CT	**Sabah Baibeche** Skikda, Algeria	Stability Result for a Weakly Nonlinearly Damped Porous System with Distributed Delay
11:40–12:00	CT	**Osman Alagoz** Bilecik, Turkey	Approximation by Sampling Durrmeyer Operators in Weighted Space of Functions
12:00–12:20	CT	**Anil Zenginoglu** Maryland, USA	Hyperboloidal Compactification for Wave Propagation
12:20–12:40	CT	**Neslihan Nesliye Pelen** Samsun, Turkey	Vector-Borne Disinformation During Disasters and Emergencies
12:40–12:45	**Break**		
Moderator	**Tahir Cosgun (tahircosgun@gmail.com)**		
12:45–13:40	IS	**Volker Mehrmann** Berlin, Germany	Port-Hamiltonian Systems: A Universal Model Class for Numerical Simulation and Control and their Algebraic and Geometric Properties
Moderator	**Figen T. Mutlu (figent@eskisehir.edu.tr)**		
13:40–14:00	CT	**Manoj Kumar Patel** Nagaland, India	Characterizations of FI-semi Injective Modules

14:00–14:20	CT	**Karima Chatouh** Banta, Algeria	Linear Codes over $R = R_1R_2R_3$ and Their Applications in Secret Sharing Schemes
14:20–14:40	CT	**Ozcan Yazici** Ankara, Turkey	The Transfinite Diameter of Bernstein Sets
14:40–15:00	CT	**Nusret Karaaslan** Rize, Turkey	On Gaussian Jacobsthal-Padovan Numbers
15:00–15:20	CT	**Burak Ekici** Ankara, Turkey	Formal Categorical Reasoning
15:20–15:25	**Break**		
Moderator	**Tahir Cosgun (tahircosgun@gmail.com)**		
15:25–15:45	CT	**Gulnur C. Kizilkan** Konya, Turkey	New Computation Method of Minimum Dwell Time for The Schur Stability of Discrete- Time Linear Systems
15:45–16:05	CT	**Koushik Das** Taki, India	Set-Valued Minimax Fractional Programming Problems via Second-order Contingent Epiderivative
16:05–16:25	CT	**Melis Z. Gorgulu** Eskisehir, Turkey	High Degree B-Spline Algorithms for Numerical Solution of the Caudrey-Dodd-Gibbon Equation
16:25–16:45	CT	**Vandana Agarwal** Meerut, India	Heat Transfer in Magneto hydrodynamics Micropolar Jeffery Fluid flow through Porous Medium over an Exponentially Stretching Sheet with Thermal Radiation
16:45–17:05	CT	**Ali Demirci** Istanbul, Turkey	Stability Analysis of a Modified Holling-Tanner Model with Delay
17:05–17:10	**Break**		
Moderator	**Tahir Cosgun (tahircosgun@gmail.com)**		
17:10–18:05	IS	**Gilbert Strang** Cambridge, USA	Linear Algebra and Deep Learning
Moderator	**Ramazan Yasar (ryasar@hacettepe.edu.tr)**		
18:05–18:25	CT	**Gul Tug** Trabzon, Turkey	Accretive Surface Growth by using Bézier Curve Fitting
18:25–18:45	CT	**Ozgur Keskin** Ankara, Turkey	Closed Space-like Ruled Surfaces thanks to Rotation Minimizing Frames
18:45–19:05	CT	**T. Meryem Baran** Kayseri, Turkey	Closedness, Sobriety, Connectedness, and Compacteness in the Category of Constant Limit Spaces
19:05–19:25	CT	**Ayse Nur Altuntas Tankul** Karabuk, Turkey	A New Hypercube-like Graph
19:25–19:45	CT	**Ayla Erdur Kara** Tekirdag, Turkey	Investigations on Singular Minimal Hypersurfaces in Lorentz Space
19:45–19:50	**Break**		
Moderator	**Talha Arikan (talhapasa@gmail.com)**		

19:50–20:10	CT	**Shikha Patel** Patna, India	Structure of $F_q\mathfrak{R}-$linear (Θ, Δ_Θ)-cyclic codes
20:10–20:30	CT	**Khadidja Sabri** Oran, Algeria	Multiplicity of Solutions for a Nonhomogeneous Elliptic Equations with Critical Exponent
20:30–20:50	CT	**Werry Febrianti** West Java, Indonesia	Solving Black-Scholes Equations for Barrier Option Pricing using Adaptive Differential Evolution Neural Network Algorithm
20:50–21:10	CT	**Ercan Duzgun** Bursa, Turkey	Comparison of Bezout Elimination Method and a New Method for Solving Nonlinear Systems of Equations of Second Order
21:10–21:15	**Break**		
Moderator	**Edanur Tastan (edanurt@eskisehir.edu.tr)**		
21:15–21:35	CT	**Mohd Aquib** New Delhi, India	A Geometric Inequality for Warped Product Slant Submanifolds in Locally Metallic Product Space Forms
21:35–21:55	CT	**Selahattin Aslan** Ankara, Turkey	Each Ruled Surface Can Be Generated by a Real Quaternion
21:55–22:15	CT	**Embarka Remli** Mascara, Algeria	F-Harmonic Maps and a Proper Homothetic Vector Field
22:15–22:35	CT	**Selcen Yuksel Perktas** Adiyaman, Turkey	On a Type of Lightlike Submanifold of Poly-Norden Semi-Riemannian Manifolds
22:35–22:40	**Closing**		

Wednesday, February 02

08.00–08:05	**Opening**		
Moderator	**Figen T. Mutlu (figent@eskisehir.edu.tr)**		
08:05–09:00	IS	**Efim Zelmanov** San Diego, USA	On Groups with Torsion
Moderator	**Hande Uslu (usluh@yildiz.edu.tr)**		
09:00–09:20	CT	**Yasin Kucukarikan** Yozgat, Turkey	On Bour's Theorem of Surfaces
09:20–09:40	CT	**Mancho Manev** Plovdiv, Bulgaria	Ricci-like Solitons with Arbitrary Potential and Gradient Almost Ricci-like Solitons on Sasaki-like Almost Contact B-metric Manifolds
09:40–10:00	CT	**Bayour Benaoumeur** Mascara, Algeria	Ricci Solitons on Sasakian Manifolds Under A New Deformation
10:00–10:20	CT	**Mohammed Abdelmalek** Tlemcen, Algeria	Some Results for Constant H_r Hypersurfaces Embedded in Pseudo-Riemannian Manifolds
10:20–10:40	CT	**Anuj Kumar** Himachal Pradesh, India	Curvature Equality and Inequality for Sequential Warped Product Submanifolds
10:40–10:45	**Break**		
Moderator	**Hande Uslu (usluh@yildiz.edu.tr)**		
10:45–11:40	IS	**Tekin Dereli** Istanbul, Turkey	Connection and Curvature on the Principal Fibre Bundles over Spacetime and Gauge Field Theories of Fundamental Forces of Nature
Moderator	**Ugurcan Eser (f6520005@std.yildiz.edu.tr)**		
11:40–12:00	CT	**Halis Can Koyuncuoglu** Izmir, Turkey	(T, λ)-Periodicity on Time Scales and Massera's Theorem
12:00–12:20	CT	**Giuseppe Di Fazio** Catania, Italy	Gradient Estimates for Weak Solutions of Elliptic PDEs
12:20–12:40	CT	**Debasis Haldar** Odisha, India	Multiframelet and Multiframelet Set on $\mathbb{Q}_p$
12:40–13:00	CT	**Ali Bashan** Zonguldak, Turkey	Numerical Computation of the Nonlinear Equal Width Wave (EW) Equation Arising in Nonlinear Media
13:00–13:20	CT	**Anabela Silva** Aveiro, Portugal	Existence and Stability of Solutions for a Class of Fractional Boundary Value Problems
13:20–13:25	**Break**		
Moderator	**Figen T. Mutlu (figent@eskisehir.edu.tr)**		
13:25–13:45	CT	**Mehsin Jabel Atteya** Baghdad, Iraq	Homogeneralized (σ, τ)-Derivations of Associative Rings
13:45–14:05	CT	**Ismail Demir** Usak, Turkey	Classification of Odd-Nilpotent Leibniz Algebras

14:05–14:25	CT	**Yusuf Alagoz** Siirt, Turkey	Modules With Respect to Min-purity
14:25–14:45	CT	**Suleyman Aydinyuz** Denizli, Turkey	The Moore-Penrose Generalized Inverse of the Fibonacci Matrix and Applications to the Cryptology
14:45–15:05	CT	**Marjan Sheibani Abdolyousefi** Semnan, Iran	The Group Inverse of a Block Complex Matrices
15:05–15:10	**Break**		
Moderator	**Tahir Cosgun (tahircosgun@gmail.com)**		
15:10–15:30	CT	**Cisem Gunes Aktas** Kayseri, Turkey	Real Structures in Singular Models of $K3$-Surfaces
15:30–15:50	CT	**El hendi Hichem** Bechar, Algeria	Harmonic Maps Between Berger-Type Deformed Sasaki Metric
15:50–16:10	CT	**Rafael López** Granada, Spain	Singular Minimal Surfaces and Perfect Domes in Architecture
16:10–16:30	CT	**Metin Turgay** Konya, Turkey	Approximation by Sampling Kantorovich Series in Weighted Space of Functions
16:30–16:50	CT	**Beldjilali Gherici** Mascara, Algeria	3-dimensional C12-Manifolds
16:50–16:55	**Break**		
Moderator	**Ramazan Yasar (ryasar@hacettepe.edu.tr)**		
16:55–17:15	CT	**Bouchra Merdji** Mascara, Algeria	New Type of Metrics Deformations and Applications to Biharmonic Maps
17:15–17:35	CT	**Ibrahim Gokcan** Artvin, Turkey	An Investigation on the Vertices of $F_{u,N}$ Obtained from Lucas Numbers in Hyperbolic Geometry
17:35–17:55	CT	**Aybala Sevde Ozkapu** Konya, Turkey	A Result for F-Contraction on Orthogonal Metric Space
17:55–18:15	CT	**Pablo Gómez-Nicolás** Santander, Spain	Induced Structures on Generalized Geometry
18:15–18:20	**Break**		
Moderator	**Ugurcan Eser (f6520005@std.yildiz.edu.tr)**		
18:20–18:40	CT	**Hristo Manev** Plovdiv, Bulgaria	Para-Ricci-like solitons with arbitrary potential on para-Sasaki-like Riemannian Π-manifolds
18:40–19:00	CT	**Mohd Vasiulla** New Delhi, India	Application of Mixed Generalized Quasi-Einstein Spacetimes in General Relativity
19:00–19:20	CT	**Birojit Das** Warangal, India	Exploration of Pre-open Sets in a Fuzzy Bitopological Space via Operation Approach
19:20–19:40	CT	**Adel Delloum** Mascara, Algeria	New Methods of Construction for Ricci-Solitons
Moderator	**Talha Arikan (talhapasa@gmail.com)**		

19:40–20:00	CT	**Hamzaoui Yamina** Tlemcen, Algeria	Positive Solution for Critical Elliptic System With Neumann Condition
20:00–20:20	CT	**Dhananjay Kumar** Bihar, India	Four Classes of Perfect Nonlinear Functions
20:20–20:40	CT	**Sinem Simsek** Kirklareli, Turkey	A Block Quaternion GMRES Method for Linear Matrix Equations
20:40–21:00	CT	**Tulay Yildirim** Karabuk, Turkey	Linear Skew Cyclic Codes over F_pR
21:00–21:20	CT	**Cemil Karacam** Istanbul, Turkey	Statistical Variants of Analogs of Korovkin Type Theorems in Banach Function Spaces
21:20–21:25	**Break**		
Moderator	**Edanur Tastan (edanurt@eskisehir.edu.tr)**		
21:25–21:45	CT	**Ebru Aydogdu** Kocaeli, Turkey	COPRAS Method Based on Entropy Measure under Complex Spherical Fuzzy Information
21:45–22:05	CT	**Fatma Karaca** Istanbul, Turkey	A Note on Gradient Ricci-Harmonic Solitons
22:05–22:25	CT	**Semsi Eken Meric** Mersin, Turkey	Some Properties of Curvatures on the Total Manifold of a Riemannian Submersion
22:25–22:45	CT	**Elif Guner** Kocaeli, Turkey	Spherical Fuzzy Sets: AHP-COPRAS Method Based on Hamacher Aggregation Operator
22:45–23:05	CT	**Zegga Kaddour** Mascara, Algeria	From a Single Sasakian Manifold to a Family of Sasakian Manifolds
23:05–23:10	**Closing**		

Thursday, February 03

08.00–08:05	**Opening**		
Moderator	**Hande Uslu (usluh@yildiz.edu.tr)**		
08.05–09:00	IS	**Ngaiming Mok** Hong Kong, China	Holomorphic Isometric Embeddings of the Poincaré Disk
Moderator	**Hande Uslu (usluh@yildiz.edu.tr)**		
09:00–09:20	CT	**Sayooj Aby Jose** Karaikudi, India	Stability Analysis and Comparative Study on Different Eco-epidemiological Models: Stage Structure for Prey and Predator Concerning Impulsive Control
09:20–09:40	CT	**Zehra Ozdemir** Amasya, Turkey	A Kinematic Model of the Rytov's Law in the Optical Fiber via Split Quaternions: Application to Electromagnetic Theory
09:40–10:00	CT	**Soumia Saidi** Jijel, Algeria	A Differential Inclusion of Second-Order and Application to Control
10:00–10:20	CT	**Alper Vural** Istanbul, Turkey	N-dimensional Lattice Path Enumeration
10:20–10:40	CT	**Ibrahim Demir** Istanbul, Turkey	Correlation Coefficients of Interval-Valued Fermatean Hesitant Fuzzy Sets with Their Application
10:40–10:45	**Break**		
Moderator	**Ugurcan Eser (f6520005@std.yildiz.edu.tr)**		
10:45–11:05	CT	**Rashid Abu-Dawwas** Irbid, Jordan	On Graded 1-Absorbing Primary Ideals
11:05–11:25	CT	**Asma Guemoula** ElOued, Algeria	Some Inequalities For Bounded Linear Operators In The New Norm On B(H)
11:25–11:45	CT	**Samet Sarioglan** Ankara, Turkey	Freeness of Generalized Spline Modules on Cycles over Bivariate Polynomial Rings
11:45–12:05	CT	**Firdous Ahmad Mala** Mohali, India	When Enumeration Becomes an Algebraic Concern
12:05–12:25	CT	**Mustafa Soyertem** Usak, Turkey	Reduction of Exhausters by Set Order Relations and Cones
12:25–12:30	**Break**		
Moderator	**Figen T. Mutlu (figent@eskisehir.edu.tr)**		
12:30–13:25	IS	**Niovi Kehayopulu** Kocaeli, Turkey	On Hypersemigroups
Moderator	**Edanur Tastan (edanurt@eskisehir.edu.tr)**		
13:25–13:45	CT	**Fatma Muazzez Simsir** Konya, Turkey	In Memoriam Cem Tezer
13:45–14:05	CT	**Sinacer Moulay Larbi** Mascara, Algeria	3-dimensional C_9-Manifolds

14:05–14:25	CT	**Abdelli Manel** Skikda, Algeria	Well-Posedness and Exponential Stability of Porous- Elastic System with Microtemperature
14:25–14:45	CT	**Nazli Irkil** Diyarbakir, Turkey	On Behavior of Solutions to A Logarithmic Wave Equation with Nonlinear Dynamical Boundary Conditions
14:45–15:05	CT	**M. Younus Bhat** Pulwama, India	Wigner-Ville Distribution associated with the Quaternion Quadratic-Phase Fourier Transform
15:05–15:25	CT	**Abir Kicha** Jijel, Algeria	New Stability Criteria of Nonlinear Non-Autonomous Systems
15:25–15:30	**Break**		
Moderator	**Talha Arikan (talhapasa@gmail.com)**		
15:30–15:50	CT	**Harun Selvitopi** Erzurum, Turkey	Numerical Investigation of Damped Wave Type MHD Flow with Time-Varied External Magnetic Field
15:50–16:10	CT	**I.Medjadj** Oran, Algeria	Global Existence Results for Second Order Neutral Functional Differential Equation with State-Dependent Delay
16:10–16:30	CT	**Ibtissem Daira** Souk Ahras, Algeria	Study on Time Scale Positive Periodic Solutions for Two Kinds of Non-linear Neutral Dynamic Equations with Vatiable Delay
16:30–16:50	CT	**Nilay Tuna** Ankara, Turkey	A Study on Dual-Generalized Complex Fibonacci p-Numbers
16:50–16:55	**Break**		
Moderator	**Hande Uslu (usluh@yildiz.edu.tr)**		
16.55–17:50	IS	**Jun Zhang** Ann Arbor, USA	Information Geometry and Statistical Mirror Symmetry
17:50–17:55	**Break**		
Moderator	**Ramazan Yasar (ryasar@hacettepe.edu.tr)**		
17:55–18:15	CT	**Esref Erdogan** Konya, Turkey	Fixed Point Results for Almost F-Type Contraction
18:15–18:35	CT	**Davood Seifipour** Abadan, Iran	On the Geometry of Lift Metrics and Lift Connections on the Tangent Bundle
18:35–18:55	CT	**Nour Elhouda Djaa** Relizane, Algeria	On the Geodesics of Deformed Sasaki Metric
18:55–19:00	**Break**		
Moderator	**Figen T. Mutlu (figent@eskisehir.edu.tr)**		
19:00–19:20	CT	**Suman Sarkar** Odisha, India	Analysis of Boundary Layer Flow over a Moving Plate in a Porous Medium
19:20–19:40	CT	**Gaukhar Shaikhova** Nur-Sultan, Kazakhstan	Analytical Methods in the Theory of Nonlinear PDEs: Two-dimensional Hirota System of Equations

19:40–20:00	CT	**Alfi Y. Zakiyyah** Jakarta, Indonesia	An Application of Graf Neural Networks in Diabetes Mellitus Protein Interactions
20:00–20:20	CT	**Mustafa Altin** Bingol, Turkey	Weighted Minimal Translation Surfaces According to Semi-Symmetric Connections in Euclidean and Minkowskian 3-Space with Density
20:20–20:40	CT	**Yahya Rahrovi** Tabriz, Iran	Hybrid CAS and Block Pulse Functions for Numerical Solution of Fractional Population Growth Model
20:40–20:45	**Break**		
Moderator	**Tahir Cosgun (tahircosgun@gmail.com)**		
20:45–21:05	CT	**Bedia Merih Ozcetin** Istanbul, Turkey	On the Intersection Curve of Two Surfaces in E^n
21:05–21:25	CT	**Ibrahim Unal** Ankara, Turkey	Geometries Related to the Critical Values of Calibrations
21:25–21:45	CT	**Ali Aydogdu** Izmir, Turkey	A Novel Entropy Measure Based on Similarity Measure for Spherical Fuzzy Sets and Its Application on Multicriteria Decision Making Problems
21:45–22:05	CT	**Ramdayal Singh Kushwaha** Banasthali, India	$\mathbb{R}-$Complex Finsler Spaces with an Exponential Metric
22:05–22:25	CT	**Fatih Sirin** Istanbul, Turkey	On Basicity of the System of Eigenfunctions for one Discontinuous Operator Produced by a Spectral Problem in Banach Function Space
22:25–22:30	**Closing**		

Port-Hamiltonian Systems: A Universal Model Class for Numerical Simulation and Control and their Algebraic and Geometric Properties

Volker Mehrmann IS

Department of Mathematics, Berlin Technical University, Berlin, Germany

Most real world dynamical systems consist of subsystems from different physical domains, modeled by partial-differential equations, ordinary differential equations, algebraic equations, combined with input and output connections. To deal with such complex systems, in recent years the class of dissipative port-Hamiltonian (pH) systems has emerged as a very efficient new modeling methodology. The main reasons are that the network based interconnection of pH systems is again pH, Galerkin projection in PDE discretization and model reduction preserve the pH structure and the physical properties are encoded in the geometric properties of the flow as well as the algebraic properties of the equations. Furthermore, dissipative pH system form a very robust representation under structured perturbations and directly indicate Lyapunov functions for stability analysis.

We discuss dissipative pH systems and describe, how many classical models can be formulated in this class. We illustrate some of the nice algebraic properties, including local canonical forms, the formulation of an associated Dirac structure, and the local invariance under space-time dependent diffeomorphisms.

The results are illustrated with some real world examples.

On Groups with Torsion

Efim Zelmanov

IS

Department of Mathematics, University of California, San Diego, USA

We will review old and new results related to the celebrated Burnside problem of 1902 if all finitely generated torsion groups are finite.

Holomorphic Isometric Embeddings of the Poincaré Disk

Ngaiming Mok IS

Department of Mathematics and Physics, Chinese Academy of Sciences, Hong Kong, China

Let $f : (\Omega; \lambda ds_\Omega) \to (\Omega'; ds_{\Omega'})$ be a germ of holomorphic isometry up to a normalizing positive constant $\lambda \in \mathbb{R}$ with respect to the Bergman metric from a bounded domain $\Omega \Subset \mathbb{C}^n$ to a bounded domain $\Omega' \Subset \mathbb{C}^{n'}$. Under the assumptions that the Bergman kernels $K_\Omega(z, w)$ and $K_{\Omega'}(\zeta, \xi)$ are rational functions in $(z, \overline{w})$ resp. $(\zeta, \overline{\xi})$, and that the Bergman metrics ds_Ω and $ds_{\Omega'}$ are complete, in an article in *J. Eur. Math. Soc.* (2012), we proved that f must extend to a proper holomorphic isometry $F : \Omega \to \Omega'$ up to a normalizing constant such that $\mathrm{Graph}(F) \subset \Omega \times \Omega'$ extends to an irreducible affine algebraic subvariety $V \subset \mathbb{C}^n \times \mathbb{C}^{n'}$. This applies in particular to the case where Ω is the unit disk Δ and $\Omega' \Subset \mathbb{C}^{n'}$ is a bounded symmetric domain in its Harish-Chandra realization. Many examples of nonstandard holomorphic isometries from the Poincaré disk to a bounded symmetric domain were constructed by Mok (2012, 2017) and by others, and some structural results have been established in the cases where Ω' is a polydisk by Ng (2010) and Chan (2016). In a recent article of Chan-Mok in *J. Differential Geom.* (2022), we proved that any such holomorphic isometry must be asymptotically totally geodesic at a general point of the unit circle. As an application, we deduce that any equivariant holomorphic embedding from a bounded symmetric domain Ω to a bounded symmetric domain Ω' must be totally geodesic, extending by a completely different method a result of Clozel (2007) proven in the cases where all irreducible factors are classical domains using arguments from Lie theory. We deduce therefrom a uniformization theorem ascertaining that given any irreducible algebraic subset Z of a bounded symmetric domain Ω in its Harish-Chandra realization, and a discrete subgroup $\Gamma \subset \Omega$ such that Γ stabilizes Z and Z/Γ is compact (hence projective), $Z \subset \Omega$ must necessarily be totally geodesic, extending in the projective case a result of Ullmo-Yafaev under the assumption that Γ is a subset of an *arithmetic* lattice $\Gamma_0 \subset \mathrm{Aut}(\Omega)$, for the proof of which the arithmeticity assumption was used in an essential way.

Non-finitely Based and Limit Varieties of Algebraic Systems

Vesselin S. Drensky IS

Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, Sofia, Bulgaria

A variety of algebraic systems is non-finitely (or infinitely) based if it does not have a finite basis of identities. It is limit (just-non-finitely based) if it is non-finitely based but all its proper subvarieties are finitely based. By the Zorn lemma every variety without a finite basis of identities contains a limit subvariety.

We survey results on non-finitely based varieties of groups, semigroups and on associative, Lie and nonassociative rings and algebras. We also present new examples of limit varieties of nonassociative algebras and varieties of pairs over fields of characteristic 0.

Although the problems are stated in the same way for different algebraic systems, it has turned out that the results are completely different for different classes of algebraic systems. The topics of the talk have also some relations with mathematical logic.

Information Geometry and Statistical Mirror Symmetry

Jun Zhang

IS

Department of Psychology, University of Michigan, Ann Arbor, USA

A parametric statistical model is a family of probability density functions over a given sample space, whereby each function is indexed by a parameter taking value in some subset of Rn. Treating parameterization as a local coordinate chart, the family forms a manifold M endowed with a Riemannian metric g given by the Fisher-information (the well-known Fisher-Rao metric). The classical theory of information geometry prescribes a family of dualistic, torsion-free conjugate connections constructed from Amari-Chensov tensor as deformation from the Levi-Civita connection associated with g. Here we prescribe an alternative geometric framework of the manifold M by i) treating the parameter as an affine parameter of a flat connection and then ii) prescribing its g-conjugate connection as a curvature-free but torsion-carrying one. From our new model, we are able to construct a pair of distinct objects on the tangent bundle TM based on the Sasaki lift of g and a canonical split using data from the base manifold M. The pair consists of a Hermitian structure and an almost Kahler structure simultaneously constructed that are in "mirror correspondence." In analogous to mirror symmetry in string theory between a complex manifold on the one side and a symplectic manifold on the other, we call this "statistical mirror-symmetry," and speculate its meaning in the context of statistical inference.

Acknowledgements: This study is a joint work with Gabriel Khan.

Connection and Curvature on the Principal Fibre Bundles over Spacetime and Gauge Field Theories of Fundamental Forces of Nature

Tekin Dereli

IS

Department of Mathematics, Koc University, Istanbul, Turkey

The mathematical structure that is common to all fundamental forces of nature acting among the constituent particles of matter is today well-established. The fundamental forces are due to the exchange of gauge vector bosons by quarks and leptons which are all spin-1/2 fermion fields. The prototype of a gauge field theory is Maxwell electrodynamics: Electromagnetic forces are due to the exchange of (massless, electrically neutral) photons between electrically charged objects. It is an old classical field theory with a relatively recent $U(1)_{EM}$ gauge field theoretic interpretation. We are now similarly confident with the standard $SU(2)_I \times U(1)_Y$ electroweak gauge theory among three generations of leptons that should be augmented by a Higgs mechanism $SU(2)_I \times U(1)_Y \to U(1)_{EM}$ for mass generation. Essentially in a similar way, the asymptotically free forces among colored quarks are generated by the exchange of $SU(3)_C$ gluons in Quantum Chromodynamics, QDC for short. In all of these cases we work on a principal fibre bundle

$$\begin{array}{c} G \to P \\ \downarrow \\ M \end{array}$$

with the 4-dimensional space-time as the base manifold M and the local Lie symmetry group G as the fibre. The pull-back of a linear connection on the bundle is then identified as the Yang-Mills potential 1-form A and the corresponding curvature $F = dA + \frac{1}{2}[A, A]_{Lie}$ with the Yang-Mills field 2-form.

The question here is whether the gravitational forces that act-at-a-distance among all massive objects admit a similar gauge theoretic structure. The answer is yes in a general sense, but no in some other respects. In my talk, I will briefly explain how a dynamical theory of gravitation over the 4-dimensional space-time manifold $M, g, \hat{\nabla}$ can be given as a gauge theory of the local Lorentz group $G = SO(1,3) \subset Spin(1,3) \cong SL(2, \mathbf{C})$ on the orthonormal frame bundle over space-time. I am going to emphasize the common mathematical structure while at the same time pointing out the essential differences.

On Hypersemigroups

Niovi Kehayopulu

IS

TUBITAK-TBAE, Kocaeli, Turkey

This is from the paper Hypergroupes canoniques valués et hypervalués by Mittas in Mathematica Balkanica 1971 [1]: "The concept of hypergroup introduced by Fr. MARTY in 1933 is as follows: A hypergroup is a nonempty set H endowed with a multiplication xy such that, for every $x, y, z \in H$ the following hold: $(1)xy \subseteq H$, $(2)x(yz) = (xy)z$, and $(3)xH = Hx = H$. The first condition expresses that the multiplication is an *hyperoperation* on H, in other words, the composition of two elements x, y of H is a subset of H. It is very easy to prove that for any $x, y \in H$, we have $xy \neq \emptyset$. In the above definition, condition (2) has no sense as x is an element, xya set and "." a multiplication between elements. There is no any explanation by Mittas what the notation in (2) means, and certainly there is no proof that $xy \neq \emptyset$. A nonemty set H having the properties (1) and (2) is called *hypersemigroup*. Even if we say $x(yz) = (xy)z$ means that $\cup_{u\in yz}xu = \cup_{v\in xy}vz$ (not given by Mittas), we never use this last equality in the text in which we always use the $x(yz) = (xy)z$. Operations between elements and sets are considered as the same operation in the bibliography. We can do whatever we want, but it is no possible to write a paper on this structure if we adopt this notation. In addition, all the results on hypersemigroups obtained using only sets are corollaries of corresponding results on le-semigroups (: lattice ordered semigroups having a greatest element e with respect to the order). So possible applications can be searched into the le-semigroups or ordered semigroups that are more simpler structures.

Keywords: Hyperoperation, hypersemigroup.

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Φ-variational Approximation by Mellin-type Nonlinear Integral Operators

Ismail Aslan **CT**

Department of Mathematics, Hacettepe University, Ankara, Turkey

In this study, we concentrate on the φ-variational approximation by Mellin-type operators. It is known that Mellin analysis is an important issue not only in approximation theory [1,2], but also in signal analysis, optical physics, engineering and etc. [5,6,8]. On the other hand, it is known that Musielak and Orlicz generalized the classical Jordan variation to the φ-variation, which is known as Musielak-Orlicz φ-variation in the literature [7]. In this work, we generalize Mellin-type nonlinear operators by a regular summability method defined in [4]. Then we obtain a general approximation theorem together with the error estimations under convergence in φ-variation. In this approximation, since it seems suitable for Mellin operators, differently from Bardaro, Sciamannini and Vinti in [3], we consider Haar measure instead of Lebesgue measure. In addition, by using our approximations, we give a general characterization theorem for φ-absolutely continuous functions. At the end, we illustrate our approximations by a suitably constructed Mellin operator.

Acknowledgements: This study is supported by the Scientific and Technological Research Council of Turkey (TuBITAK 3501), Grant no: 119F262.

Keywords: Convergence in φ-variation, Mellin operators, rate of approximation, summability methods, φ-absolutely continuous functions.

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Stability Result for a Weakly Nonlinearly Damped Porous System with Distributed Delay

Sabah Baibeche **CT**

Department of Mathematics, University of 20 Out 1955, Skikda, Algeria

In this paper, we consider a one-dimensional porous system damped with a single weakly nonlinear feedback and distributed delay term. Without imposing any restrictive growth assumption near the origin on the damping term, we establish an explicit and general decay rate, using a multiplier method and some properties of convex functions in case of the same speed of propagation in the two equations of the system. The result is new and opens more research areas into porous-elastic system.

Keywords: Convex functions, damping term, single weakly, porous-elastic system, general decay.

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Approximation by Sampling Durrmeyer Operators in Weighted Space of Functions

Osman Alagoz[1], ***Metin Turgay***[2], ***Tuncer Acar***[2], ***Merve Parlak***[1] **CT**

[1] Department of Mathematics, Bilecik University, Bilecik, Turkey
[2] Department of Mathematics, Selcuk University, Konya, Turkey

The present paper deals with local and global approximation behaviours of sampling Durrmeyer operators for functions belonging to weighted spaces of continuous functions. After giving some fundamental notations of sampling type approximation methods and presenting well-definity of the operators on weighted spaces of functions, we examine pointwise and uniform convergence of the family of operetors and determine the rate of convergence via weighted modulus of continuity. A quantitative Voronovskaja theorem is also studied in order to obtain rate of pointwise convergence and upper estimate for this convergence. The last section is devoted to some examples of kernel functions which satisfy the theorems obtained. Some graphical examinations are presented as well.

Keywords: Pointwise convergence, uniform convergence, weighted modulus, Voronovskaya, kernel.

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Hyperboloidal Compactification for Wave Propagation

Anil Zenginoglu **CT**

Institute for Physical Science and Technology, University of Maryland, Maryland, USA

Wave propagation problems on unbounded domains appear in various disciplines such as seismics, underwater acoustics, electrodynamics, and general relativity. The unbounded solution domain is typically truncated in numerical calculations. This artificial truncation fails to capture radiative fields defined at infinity and introduces outer boundaries that may contaminate the interior solution due to artificial reflections. Penrose compactification at null infinity uses conformal geometry to translate asymptotic analysis to local analysis. We can map infinity to a grid shell and solve wave equations numerically on the entire unbounded domain using this geometric tool. The associated hyperboloidal spacetime transformation not only eliminates problems related to radiation extraction and outer boundary treatment, but also enhances the numerical efficiency of certain interior calculations. Hyperboloidal compactification has already become instrumental in numerical relativity for computing gravitational waves around black holes. I will describe hyperboloidal compactification and its potential for computational mathematics, engineering, and technology to solve real-world problems such as the computation of radar cross-sections of airplanes or the propagation of seismic waves across large distances.

Keywords: Black holes, gravitational waves, Helmholtz equations, hyperbolic geometry, Maxwell equations.

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Vector-Borne Disinformation During Disasters and Emergencies

Neslihan Nesliye Pelen[1], ***Meltem Golgeli***[2] **CT**

[1]Department of Mathematics, Ondokuz Mayis University, Samsun, Turkey
[2]Department of Mathematics, TOBB University of Economics and Technology, Ankara, Turkey

During disasters and emergencies (earthquakes, pandemics, economic crises etc.), we also face a second challenge, pollution of information. The transmitted information may be false, potentially harmful and speculative. Today, the main source of information seems to be the social media, which behaves as a vector via sharing news. The first rumor model was studied by Daley and Kendall [1] and we will propose another rumor model that consider the several properties of the social media.

In this manuscript, the concept of the transmission dynamics of vector-borne diseases that is studied several times before like in [2] is adapted to the transmission dynamics of vector-borne disinformation. The dynamical behaviour of the model is analysed, the disinformation-free and disinformation endemic equilibria of the model are found and both their local and global stabilities are presented. Finally, numerical simulations are carried out to support the analytical results of the dynamical transmission of disinformation.

Keywords: Mathematical modeling, mathematical sociology, stability, rumor model, equilibrium.

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(T, λ)-Periodicity on Time Scales and Massera's Theorem

Halis Can Koyuncuoglu **CT**

Department of Engineering Sciences, Izmir Katip Celebi University, Izmir, Turkey

In this study, we introduce (T, λ)-periodic functions on time scales by employing shift operators $\delta_{\pm}$. In particular choices of λ, (T, λ)-periodic functions coincide with periodic, anti-periodic, or Bloch periodic functions on time scales. Thus, the proposed alternative periodicity notion does not only relaxes but also generalizes the existing periodicity definitions on time scales. Furthermore, we define a new symmetry property, namely (T, λ)-symmetry, for functions of two variables in preparation for the main results. As the major task of the research, we obtain Massera type theorems for both linear and nonlinear dynamic equations on time scales. To achieve this task, first we define a new boundedness concept so-called λ-boundedness, and then we establish a linkage between the existence of λ-bounded solutions and (T, λ)-periodic solutions of dynamic equations in both linear and nonlinear cases. In our analysis, we assume that the time scale $\mathbb{T}$ is periodic in shifts $\delta_{\pm}$ which does not need to be translation invariant. Thus, outcomes of this work are a significant contribution for the literature, since they allow us to consider more general domains not restricted to $\mathbb{T} = \mathbb{R}$ or $\mathbb{T} = \mathbb{Z}$.

Keywords: Time scale, shift operator, (T, λ)-periodic, (T, λ)-symmetric, Massera's theorem.

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Gradient Estimates for Weak Solutions of Elliptic PDE's

Giuseppe Di Fazio **CT**

Department of Mathematics and Computer Science, University of Catania, Catania, Italy

Let us consider an elliptic equation of second order in variational form i.e.

$$\nabla \cdot (A(x)\nabla u) = \nabla \cdot f$$

in a bounded domain $\Omega \subset \mathbb{R}^n$ where the function f belongs to some suitable function space. The gradient estimates problem is very important both from theoretical and applied point of view.

In the Seminar we exploit what is the heart of the technique to show gradient estimates allowing the function f to belong to very general function spaces. Our technique is very flexible and allow to show existence, uniqueness and well posedness of the Dirichlet problem in several classes. As an example we will give the result in $W_0^{1,p}$.

Keywords: Elliptic equations, gradient estimates, Morrey spaces.

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Numerical Computation of the Nonlinear Equal Width Wave (EW) Equation Arising in Nonlinear Media

Ali Bashan **CT**

Department of Mathematics, Zonguldak Bulent Ecevit University, Zonguldak, Turkey

An effective approach is implemented via fixed parameters and fewer number of mesh points to obtain better numerical solutions of the equal width wave (EW) equation. Using fewer numbers of mesh points than common methods and producing high accurate results are property of differential quadrature method with contribution of finite difference method. For the time discritization finite difference method is used and efficient linearization technique namely Rubin and Graves are implemented. Then for the space discritization differential quadrature method is applied. To display the accuracy of the new method, three well-known test problems are investigated. Present numerical results are given with earlier studies existing in the literature. The performance of the combined method observed from results of all of the applications. The three invariants with relative changes of invariants are reported. Simulations of all applications are given with details. The rates of the convergence are also investigated.

Keywords: Differential quadrature method, finite difference method, solitary waves, equal width wave equation, convergence.

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Existence and Stability of Solutions for a Class of Fractional Boundary Value Problems

Anabela Silva[1], ***Luís Castro***[1] **CT**

[1]Department of Mathematics, University of Aveiro, Aveiro, Portugal

In this talk, we investigate a class of fractional four-point boundary value problems with Caputo fractional derivative. The Banach contraction principle (also known as the Banach fixed point theorem) is a powerful tool in operator theory. Based upon Banach contraction principle, we discuss the stability of a class of fractional boundary value problems. In particular, we present sufficient conditions for the Ulam-Hyers and Ulam-Hyers-Rassias stabilities of that class of nonlinear fractional boundary value problem on an appropriate space's framework.

Keywords: Banach contraction, Caputo derivative, fractional calculus, Ulam-Hyers stability, Ulam-Hyers- Rassias stability.

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Reduction of Exhausters by Set Order Relations and Cones

Mustafa Soyertem[1], ***Ilknur Atasever Guvenc***[2], ***Didem Tozkan***[2] **CT**

[1]Department of Mathematics, Usak University, Usak, Turkey
[2]Department of Mathematics, Eskisehir Technical University, Eskisehir, Turkey

The notions of upper and lower exhausters are effective tools for the study of nonsmooth functions [1]. There are many studies presenting optimality conditions in terms of exhausters for unconstrained and constrained cases [2-4]. One can observe that optimality conditions given by both proper and adjoint exhausters are related to all elements of the exhauster [3]. Moreover, in constrained case the conditions that must be provided for a particular cone determined by the point and constraint set are rather challenging to check. Thus, it is advantageous to reduce the number of sets in an exhauster for constrained case. There are some basic results on reduction of exhausters applying some techniques such that removing redundant sets, cutting out the redundant part of a set or using the concept of shadowing sets [5-7]. However, set order relations have not been used to generate a technique for the reduction of exhausters. In this work, we first consider constrained optimization problems and deal with the problem of reducing generalized exhausters of the directional derivative of the objective function. We present some results to reduce generalized lower (upper) exhausters by using set order relations $\preceq^{m_1}$ and $\preceq^{m_2}$ presented in [8], respectively. Furthermore, we show that a generalized exhauster $\mathbb{E}$ can be reduced to the set of minimal elements of $\mathbb{E}$ with respect to $\preceq^{m_1}$ or $\preceq^{m_2}$. Also, we adapt obtained results to unconstrained optimization problems. Namely, this work provides that the problem of reduction of exhausters can be regarded as a set valued optimization problem.

Keywords: Constrained optimization, generalized exhauster, nonsmooth analysis, optimization, set valued optimization.

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Study on Time Scale Positive Periodic Solutions for Two Kinds of Non-linear Neutral Dynamic Equations with Vatiable Delay

Ibtissem Daira **CT**

Department of Mathematics and Informatics, University of Souk Ahras, Souk Ahras, Algeria

The purpose of this work is to use Krasnoselskii's fixed point theorem to prove the existence of positive periodic solutions for two kinds of nonlinear neutral dynamic equations with variable delay on a time scale. We invert these equations to construct a sum of a contraction and a completely continuous map which is suitable for applying the Krasnoselskii's theorem.

Keywords: Banach space, Krasnoselskii's fixed point theorem, Ascoli-Arzela theorem, exponential function, Green's functions.

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Numerical Investigation of Damped Wave Type MHD Flow with Time-Varied External Magnetic Field

Harun Selvitopi **CT**

Department of Mathematics, Erzurum Technical University, Erzurum, Turkey

This study concerns the observation of the behavior of damped wave-type magnetohydrodynamic (MHD) flow in a sufficiently long rectangular channel having time-varied oblique magnetic field. The mathematical model of the considered problem is coupled convection-diffusion heat-type for the velocity of the fluid and convection-diffusion wave-like for the induced magnetic field [1]. To obtain the numerical simulation, we have used the finite difference method for the discretization of the temporal variable and we also have considered the Galerkin finite element method for the discretization of the spatial variable. In the numerical investigation procedure, the time-varied oblique magnetic field has been considered several functions of the time as polynomial, exponential, and trigonometric e.t.c. We have also considered the several values of the Hartmann number and several directions of the induced magnetic field. In literature, there are many studies about numerical solution of steady and unsteady magnetohydrodynamic flow problems using finite element method, boundary element method and finite difference method e.t.c. For all that, it is the first time in the literature, the damped wave type MHD flow has been considered numerically in this study. The obtained numerical results have been presented by the contour plots.

Keywords: Damped wave, FDM, FEM, MHD, time-varied external magnetic field.

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Global Existence Results for Second Order Neutral Functional Differential Equation with State-Dependent Delay

M.Benchohra[1], ***I.Medjadj***[2,3] **CT**

[1]Laboratory of Mathematics, University of Sidi-Bel-Abbés, Sidi-Bel-Abbés, Algeria
[2]Department of Mathematics, University of Science and Technology of Oran, Oran, Algeria
[3]Laboratory of Fundamental and Applied Mathematics of Oran, University of Oran, Oran, Algeria

Our aim in this work is to provide sufficient conditions for the existence of global solutions of second order neutral functional differential equation with state-dependent delay. We use the semigroup theory and Schauder's fixed point theorem.

Keywords: Functional differential equation of second order, mild solution, infinite delay, state-dependent delay fixed point, semigroup theory.

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On Graded 1-Absorbing Primary Ideals

Rashid Abu-Dawwas[1], ***Malik Bataineh***[2] **CT**

[1]Department of Mathematics, Yarmouk University, Irbid, Jordan
[2]Department of Mathematics and Statistics, Jordan University of Science and Technology, Irbid, Jordan

Since graded prime ideals have a valuable performance in the theory of graded commutative rings, there are various procedures to generalize the concept of graded prime ideals. In [1], Al-Zoubi, Abu-Dawwas and Ceken gave a generalization of graded prime ideals, called graded 2 -absorbing ideals. A proper graded ideal P of R is said to be graded 2 -absorbing if whenever $a, b, c \in h(R)$ such that $abc \in P$, then either $ab \in P$ or $ac \in P$ or $bc \in P$. On the other hand, Refai and Al-Zoubi introduced in [4] graded primary ideals. A proper graded ideal P of R is said to be graded primary if for $x, y \in h(R)$ such that $xy \in P$, then either $x \in P$ or $y \in Grad(P)$. Recall from [2]that a proper graded ideal P of R is called a graded 2 -absorbing primary ideal of R if whenever $a, b, c \in h(R)$ with $abc \in P$, then $ab \in P$ or $ac \in Grad(P)$ or $bc \in Grad(P)$. In this article, we follow [3] to introduce and study the concept of graded 1 -absorbing primary ideals of a graded commutative rings. A proper graded ideal P of R is said to be a graded 1 -absorbing primary ideal of R if whenever nonunit elements $x, y, z \in h(R)$ such that $xyz \in P$, then $xy \in P$ or $z \in Grad(P)$.Among several results, we prove that the following implications hold and none of them is reversible:

graded primary ideal $\Rightarrow$ **graded 1-absorbing primary ideal** $\Rightarrow$ **graded 2-absorbing primary ideal**.

Keywords: Graded prime ideal, graded primary ideal, graded 2 -absorbing primary ideal, graded 2 -absorbing ideal, 20 graded n -absorbing ideal.

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Some Inequalities for Bounded Linear Operators in the New Norm on $B(H)$

Asma Guemoula **CT**

Department of Mathematics, Echahid Hamma Lakhdar University of El Oued, El Oued, Algeria

The purpose of the present work is to study some important properties for the (α, β)-norm newly introduced in [4], and obtain upper bounds for the said norm, on the space of bounded linear operators on a complex Hilbert space, from which we obtain bounds for the numerical radius of operator.

Keywords: Numerical radius, bounded linear operator, operator inequalities, Hilbert space.

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Well-Posedness and Exponential Stability of Porous-Elastic System with Microtemperature

Abdelli Manel **CT**

Department of Mathematics, 20 August 1955 University of Skikda, Skikda, Algeria

In this paper, we consider a one-dimensional porous-elastic system with microtemperature acting on the porous equation. Under suitable assumptions on the weight of distributed delay, we establish the well-posedness of the system by using semigroup theory and we prove that the unique dissipation due to the microtemperature is strong enough to exponentially stabilize the system when the speeds of wave propagation are equal.

Keywords: Semigroup, Distributed delay, porous-elastic system, Lyapunov functional, general decay.

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On Behavior of Solutions to A Logarithmic Wave Equation with Nonlinear Dynamical Boundary

Nazli Irkil*, *Erhan Piskin **CT**

Department of Mathematics, Dicle University, Diyarbakir, Turkey

This work deals with the initial boundary value problem of linear wave equation with dynamic boundary condition which is including logarithmic source term and nonlinear damping term. For the wave equation with nonlinear dynamic boundary condition arising in the physical models and control systems [4]. Without logarithmic source term wave equations with dynamic boundary condition were studied by mathematicians [2,4,8,9,10,11]. In past decades, the logarithmic wave equation which is applied in many branches of physics was discussed by many author [1,3,5,6,7]. Because of these reasons we examine the effect of logarithmic nonlinearity to dynamic boundary condition. We study local existence result by using Schauder fixed point theorem. The growth of solution was obtained. As far as know, there is no work which is related with wave equation with dynamic boundary condition and logarithmic source term. These results fill in the gaps in previous studies on this type of models.

Keywords: Asymptotic behavior, blow up, dynamic boundary conditions, existence, logarithmic nonlinearity.

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3-dimensional C_9-Manifolds

Sinacer Moulay Larbi **CT**

Faculty of Sciences and Technology, University of Mascara, Mascara, Algeria

In 1960s and 1970s, the almost contact manifolds were studied as an odd dimensional counterpart of almost complex manifolds, the warped product was used to make examples of almost contact manifolds. There are different classifications of almost contact structures, in [3], D. Chinea and C. Gonzalez have defined 12 classes of almost contact metric manifolds. According to [3], Theorem 5.1, in dimension 3, these manifolds are reduced to five classes: $|C|$ class of cosymplectic manifolds, C_5 class of -Kenmotsu manifolds, C6 class of α-Sasakian manifolds, C_9-manifolds and C_{12}-manifolds.

Most of the research related to almost contact metric structures is concerned with the normal ones i.e. the first three classes. For the last two classes which can never be normal, specifically, there are only two papers on C_{12}-manifolds. In the first one [2], The authors have developed a systematic study of the curvature of the Chinea-Gonzalez class $C_5 \bigoplus C_{12}$ and obtain some classification theorems for those manifolds that satisfy suitable curvature conditions. This class is defined by using a certain function α and when this function vanishes the class $C_5 \bigoplus C_{12}$ reduces to class C_{12}.

The second paper [1], contains new results on a particular 3-dimensional C_{12}-manifolds with a class of concrete illustrative examples.

This presentation is devoted to 3-dimensional C_9-manifolds which are never normal. Firstly, we study their fundamental properties and we give concrete examples. Secondly, we investigate such structures on three dimensional Lie groups.

Keywords: Almost contact metric manifold, Lie algebra.

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Wigner-Ville Distribution Associated with the Quaternion Quadratic-phase Fourier Transform

M. Younus Bhat[1], ***Aamir H. Dar***[1] **CT**

[1]Department of Mathematical Sciences, Islamic University of Science and Technology Awantipora, Pulwama, Jammu and Kashmir, India.

The most important time-frequency tool for analyzing non-transient signal is the Wigner-Ville distribution. In this paper, we present the two-dimensional quaternion Wigner-Ville distribution associated with quadratic phase Fourier transform(WVD-QQPFT) by intervening the advantages of quaternion quadratic-phase Fourier transforms (Q-QPFT) and Wigner-Ville distribution (WVD). First,we propose the definition of the WVD-QQPFT and its relationship with the classical Wigner-Ville distribution in the quaternion setting. Next, we investigate the general properties of the newly defined WVD-QQPFT, including complex conjugate, symmetry-conjugation, nonlinearity, boundedness, reconstruction formula, Moyals formula and Plancherel formula. Finally, we propose the convolution and correlation theorems associated with WVD-QQPFT.

Keywords: Quaternion quadratic-phase Fourier transform, Winger-Ville distribution, boundedness, Moyals formula, convolution, correlation.

New Stability Criteria of Nonlinear Non-autonomous Systems

Abir Kicha **CT**

Department of Mathematics, University of Jijel, Jijel, Algeria

The question proposed in this paper is related to the study of the preservation of uniform h-stability and uniform boundedness of time-varying nonlinear differential equations with a perturbation using Gronwall's inequalities and Lyapunov's theory. Moreover, we show the linearization technique for the uniform h-stability of a nonlinear system and give necessary and sufficient conditions for the global boundedness of perturbed systems. Some examples and simulations are given to illustrate the main results.

Keywords: Global uniform boundedness, h-stabilization, linearization system, Lyapunov function, uniform h-stability.

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A Study on Dual-generalized Complex Fibonacci p-numbers

Nilay TUNA[1], ***Elif TAN***[1] **CT**

[1]Department of Mathematics, Ankara University, Ankara, Turkey

In this study, we define a new class of dual-generalized complex numbers whose components are generalized Fibonacci p-numbers and generalized Lucas p-numbers. For $p > 0$ the nth generalized DGC (dual-generalized complex) Fibonacci and Lucas p-numbers are defined by

$$\tilde{F}_{p,m}(n) = F_{p,m}(n) + F_{p,m}(n+1)J + F_{p,m}(n+2)\epsilon + F_{p,m}(n+3)J\epsilon$$

and

$$\tilde{L}_{p,m}(n) = L_{p,m}(n) + L_{p,m}(n+1)J + L_{p,m}(n+2)\epsilon + L_{p,m}(n+3)J\epsilon$$

where $F_{p,m}$ and $L_{p,m}$ are nth generalized Fibonacci and Lucas p-numbers, respectively. Here ϵ is dual unit, J is generalized complex unit, and $J\epsilon$ dual generalized complex unit. We obtain some algebraic properties of these numbers such as conjugation identities, recurrence relations, generating functions, Binet formulas, generalized Honsberger's identity, Catalan's, Casssini's identity, d'Ocagne's identity, and sum formulas. Moreover, we develop a matrix representation for these numbers which helps to derive well known Cassini's identity. Our results can be seen as a generalization of many previous works in the literature such as [1, 2, 3, 7, 8]. The main advantage of introducing the new dual-generalized complex numbers is that many dualgeneralized complex numbers with celebrated numbers such as Fibonacci, Lucas, Pell, Pell-Lucas numbers and Fibonacci, Lucas, Pell, Pell-Lucas p-numbers can be deduced as particular cases of these numbers. The results of this study could be applied for hyperbolic-generalized complex Fibonacci p-numbers and complex-generalized complex Fibonacci p-numbers.

Keywords: Dual-generalized complex numbers, Fibonacci numbers, Lucas numbers, Fibonacci p-numbers, Lucas p-numbers.

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Characterizations of FI-semi Injective Modules

Manoj Kumar Patel **CT**

Department of Mathematics, National Institute of Technology Nagaland, Nagaland, India

In this paper, we explore some beautiful property of FI-semi injective modules and FI-self p-injective ring. Also, discussed various characterizations of FI-semi injective modules in terms of their endomorphism rings. Finally, we prove that the property of being FI-semi injective of a module is Morita invariant.

Keywords: Endomorphism rings, principally injective rings and modules, semi-injective modules.

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Linear Codes over $R = R_1R_2R_3$ and Their Applications in Secret Sharing Schemes

Karima Chatouh **CT**

Department of Mathematics, Mostefa Ben Boulaïd University, Banta, Algeria

The main objective is to provide an important overview of information security. The survey argues that cryptographic techniques are the most effective techniques for protecting high-value data. A secret sharing schemes is a significant concept in cryptography. For this, several authors have examined certain linear codes and have classified the access structure of secret sharing schemes from their dual codes. However, it is a complicated problem to determine the access structure of the secret sharing schemes based on a linear code over finite rings. Thereby, the first part of this work will focus on the one hand, on the construction of linear codes over finite ring $R = R_1R_2R_3$. Also, we determine secret sharing schemes based on linear codes over finite rings $R = R_1R_2R_3$ ant their access structures. The second part will concern the applications of linear code over finite ring $R = R_1R_2R_3$ in the secret sharing scheme, we will describe the most important aspects related to the innovative procedures of the secret sharing schemes.

Keywords: Linear codes, simplex and MacDonald codes, sharing schemes, gray image, gray map.

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On Bour's Theorem of Surfaces

Yasin Kucukarikan[1], ***Burcu Bektas Demirci***[2], ***Murat Babaarslan***[1] **CT**

[1]Department of Mathematics, Bozok University,Yozgat, Turkey
[2]Department of Civil Engineering, Fatih Sultan Mehmet Vakif University, Istanbul, Turkey

In 1862, E. Bour gave a well known result about an isometry between a helicoidal surface and a rotational surface in 3-dimensional Euclidean space as follows. Bour's Theorem [2]. A helicoidal surface is isometric to a rotational surface so that helices on the helicoidal surface correspond to parallel circles on the rotation surface [2]. D. T. Hieu and N. N. Thang studied Bour's theorem by defining the helicoidal surface in 4-dimensional Minkowski space (also called as Minkowski space-time). In addition, using the isometric surfaces according to Bour's theorem having the same Gauss map in 4-dimensional Minkowski space, they investigated geometric properties of surfaces such as hyperplanar and minimal [3]. Moreover, M. Babaarslan and N. Sonmez [1] gave the parametrizations of three types of helicoidal surfaces obtained from rotations with 2-dimension axes in 4-dimensional Minkowski space. In this work, we will study on Bour's theorem for the three types of helicoidal surfaces defined in [1]. Also we will obtain geometric properties of isometric surface pairs such as hyperplanar and minimal by using the Gauss map.

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Keywords: Rotational Surfaces, helicoidal surfaces, isometry, Gauss map, 4-dimensional Minkowski space.

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The Transfinite Diameter of Bernstein Sets

Ozcan Yazici **CT**

Department of Mathematics, Middle East Technical University, Ankara, Turkey

A compact set $K \subset \mathbb{C}^n$ is called Bernstein set if, for some constant $M > 0$, the following inequality

$$\|D^\alpha P\|_K \leq M^{|\alpha|}(degP)^{|\alpha|}\|P\|_K$$

is satisfied for every multiindex $\alpha \in \mathbb{N}^n$ and for every polynomial P. It is known that a compact set K is pluripolar if and only if its transfinite diameter $d(K) = 0$. Thus finding a lower bound for transfinite diameter of compact sets is an essential problem in pluripotential theory. We provide here a lower bound for the transfinite diameter of Bernstein sets by using extremal set of points which was introduced in [4].

Keywords: Bernstein sets, Fekete points, Markov sets, pluripolar Sets, pluripotential theory, transfinite diameter.

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On the Geodesics of Deformed Sasaki Metric

Nour El Houda Djaa **CT**

Department of Mathematics, Relizane University, Relizane, Algeria

In 1958 Sasaki has discovered on the tangent bundle TM a new canonical almost hermitian metric, since then many authors have largely devoted their studies to this topic. Among them we can mention Dombrowski [3], Opriou [6], Yano and Ishihara [5, 9], etc...

The stiffness of the Sasaki metric has incited a number of authors to deform the Sasaki metric in order to achieve a kind of flexibility of its properties. (see [1, 2, 4, 8, 10] and others). In this paper, we consider the tangent bundle TM over a Riemannian manifold endowed with a deformed Sasaki metric g_{DS} like a new distorted metric on TM by using a vertical deformation of Sasaki metric. First we present the geometric result concerning the connection of Levi-Civita and all forms of Riemannian curvature tensors of this metric. Secondly, we study the geodesics on the tangent bundle TM and unit tangent bundle T_1M. Finally, we characterize the geodesic curvatures on T_1M. Our perspective in forthcoming works is, the characterization of the harmonic and biharmonic vector fields with respect to the deformed Sasaki metric g_{DS}. Study the harmonicity of the different canonical projections and inclusions of (TM, g_{DS}) and we search the harmonicity of the deformed Sasaki metric g_{DS} and the Sasaki metric g_S with respect to each other. In the second step, we study the geometry of deformed sasaki metric over a kahlerian manifold (M, g, ϕ).

Keywords: Curvatures tensors, vertical lift, horizontal lift, tangent bundle, unit tangent bundle.

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On Gaussian Jacobsthal-Padovan Numbers

Nusret Karaaslan **CT**

Kalkandere Dagdibi Secondary School, Ministry of National Education, Rize, Turkey

Gaussian Jacobsthal-Padovan numbers have been the central focus of this paper and firstly this number sequence has defined. Later, we have given the proof of the generating function of the Gaussian Jacobsthal-Padovan sequence. After that by using generating function, we have given the proof of the Binet formula for this number sequence. Additionaly, we have investigated some properties such as Simson identity, summation formulas of this sequence. Finally, we have obtained some matrices whose elements are Gaussian Jacobsthal-Padovan numbers.

Keywords: Binet formula, generating function, Jacobsthal numbers, Jacobsthal-Padovan numbers, recurrence relation.

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Formal Categorical Reasoning

Burak Ekici **CT**

Computer Engineering Department, TED University, Ankara, Turkey

Category theory, among other capabilities, serves as a domain language of interpretation for various sorts of computational calculi [1]. For instance, the category induced by any Cartesian closed preorder speaks the same formal language with that of Intuitionistic Propositional Logic (IPL). Similarly, Simply Typed Lambda Calculi can be viewed as the internal language of Cartesian closed categories [2, 3]. Monads [4] thus adjunctions have been benefited by and large when it comes to capture computational side effects, and implemented in Haskell language [5]. Besides, its structural nature allows for formalization of several categorical notions along with many property proofs in type theory based proof assistants such as Coq [6], Lean and Agda. In this talk, I briefly introduce a category theory library implemented in the Coq proof assistant. The library [7] is centered around objects that help modeling programming language constructs. This can be seen as the first step towards providing some formal recipe to study programming language semantics. I conclude with IPL soundness proof in a categorical setting as a use case and/or application to the Coq library.

Keywords: Categorical logic, denotational semantics of programming constructs, formal proofs, the Coq proof assistant.

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Homogeneralized (σ, τ)-Derivations of Associative Rings

Mehsin Jabel Atteya **CT**

Department of Mathematics, Al-Mustansiriya University, Baghdad, Iraq

The commutativity of associative rings with derivations has become one of the focus points of several authors and significant work has been done in this direction.

The study of derivation was initiated during the 1950s and 1960s. Derivations of rings got a tremendous development in 1957 when Posner [1] established two very striking results in the case of prime rings. In 2000, El Sofy [2] defined a homoderivation on R as an additive mapping $h : R \to R$ satisfying $h(xy) = h(x)h(y) + h(x)y + xh(y)$ for all $x, y \in R$. Precisely, we prove the commutativity with other cases of a ring that satisfied certain conditions and introduce the definition of Homogeneralized (σ, τ)-derivations via associative ring as following:

Definition: Let R be a ring and σ, τ be automorphism mappings of R. An additive mapping $H : R \to R$ is called a Homogeneralized (σ, τ)-derivation of R if $H(xy) = H(x)H(y)+H(x)\sigma(y)+\tau(x)h(y)$ for all $x, y \in R$, where $h : R \to R$ is (σ, τ)-Homoderivation of R.

Keywords: Homoderivation, semiprime rings, strong commutativity-preserving mappings, weakly semiprime ideal, weak zero-divisor.

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Classification of Odd-nilpotent Leibniz Algebras

Ismail Demir **CT**

Department of Mathematics, Usak University, Usak, Turkey

Leibniz algebras are certain generalization of Lie algebras. Classification of all nilpotent Lie algebras is still unsolved and only the complete classifications of nilpotent Lie algebras over an algebraically closed field up to dimension 7 are known [1]. The problem of classifying all nilpotent Leibniz algebras is more complicated due to lack of antisymmetry property. The complete classification of 4-dimensional complex nilpotent Leibniz algebras given up to this date [4] and researchers focus on classifying subclasses of nilpotent Leibniz algebras on higher dimensions [5]. When dimensions of the ideals in lower central series are all odd numbers we name this particular subclass of nilpotent Leibniz algebras as odd-nilpotent Leibniz algebras. Using the canonical forms for the congruence classes of matrices of bilinear forms given in [2], we classify the aforementioned subclasses of nilpotent Leibniz algebras up to dimension 7. We use Mathematica program implementing Algorithm 2.6 given in [3] to check that the classes we obtained are indeed pairwise nonisomorphic.

Keywords: Antisymmetry, bilinear forms, congruence classes, Lie algebras, lower central series.

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Modules with Respect to Min-purity

Yusuf Alagoz[1], ***Ali Moradzadeh-Dehkordi***[2] **CT**

[1]Department of Mathematics, Siirt University, Siirt, Turkey
[2]Department of Sciences, University of Isfahan, Isfahan, Iran

Min-injective rings and min-injective modules has a substantial role in module-ring theory and representations of algebras. Numerous characterizations of Quasi-Frobenius rings were given by various authors. Weakening the injectivity condition on Ikeda's [6] famous characterization, the concept of min-injectivity for rings appeared as an important property for characterizing Quasi-Frobenius rings. It was shown that a ring R is Quasi-Frobenius if and only if it is two-sided Artinian and two-sided min-injective [9]. As a generalization of injectivity, Harada [4] introduced the concept of min-injective modules. In [7], Mao introduced the concept of min-pure exact sequences, which was used to characterize min-injective modules and to investigate the existence of min-injective covers. The interest on min-injective covers on the one hand, and min-injective modules on the other hand show that the investigation of the problems in this subject from the min-purity view will be effective. In this project, our aim is to identify and study most significative objects of min-purity: min-pure injective and projective objects. Along the way, using the concept of min-purity, we present some characterizations of 'universally mininjective', 'Quasi-Frobenius', 'semisimple' and 'Artinian' rings. The relations between the aforementioned objects and other well-known homological objects are investigated. Mainly we are interested in characterizing the rings whose min-pure injective (resp. projective) modules are injective (resp. projective) and the rings whose min-pure injective (resp. projective) modules are projective (resp. injective). Also, the structure of the rings whose injective modules are min-pure projective are considered. Finally, we consider the rings whose all modules are min-pure injective (resp. projective).

Keywords: Universally mininjective rings, min-pure submodules, min-pure injective modules, min-pure projective modules, quasi-Frobenius rings.

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The Moore-Penrose Generalized Inverse of the Fibonacci Matrix and Applications to the Cryptology

Suleyman Aydinyuz[1], ***Mustafa Asci***[1] **CT**

[1]Department of Mathematics, Pamukkale University, Denizli, Turkey

In this paper, we define and study the Moore-Penrose generalized inverse of the rectangular or square Fibonacci matrices. Then, we introduce a new encryption method. In this method, we get encryption using the Moore-Penrose generalized inverse of the Fibonacci matrix. We give illustrative examples about this encryption method. The singular value decomposition (SVD) of a matrix has very important role in computation to the Moore-Penrose generalized inverse of a rectangular matrix. In this method, we use the square or rectangular Fibonacci matrix which has the Moore-Penrose generalized inverse as a key matrix. Therefore, with this method, we can encrypt the message text of any length we want. It can also be implemented by rectangular matrices without the need for square matrices when performing encryption.

Keywords: Fibonacci matrix, the Moore-Penrose generalized inverse, encryption, singular value decomposition (SVD), cryptology.

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The Group Inverse of a Block Complex Matrices

Marjan Sheibani Abdolyousefi **CT**

Department of Mathematics, Semnan University, Semnan, Iran

Let A be a complex Banach algebra with an identity 1. An element a in a Banach algebra A has group inverse provided that there exists $b \in A$ such that $ab = ba$, $b = bab$, and $a = aba$. Such b is unique if exists, denoted by $a^{\#}$, and called the group inverse of a. As it is well known, a square complex matrix A has group inverse if and only if $rank(A) = rank(A^2)$. In this paper, we investigate the group inverse of 2×2 block complex matrix

$$M = \left(\begin{bmatrix} A & B \\ C & D \end{bmatrix} \right),$$

where $A \in \mathbb{C}^{m\times m}, B \in \mathbb{C}^{m\times n}, C \in \mathbb{C}^{n\times m}, D \in \mathbb{C}^{n\times n}$. We establish necessary and sufficient conditions for which M has a group inverse. The explicit formula for $M^{\#}$ is thereby given. In [1, Theorem 3.4 and Theorem 3.6], Beni'tez et al., investigated certain sufficient conditions for which M has group inverse. We find some new conditions under which M has group inverse and extend their theorems as corollaries of our main result.

Keywords: Banach algebra, block matrix, Drazin inverse, group inverse, spectral idempotent.

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Freeness of Generalized Spline Modules on Cycles over Bivariate Polynomial Rings

Samet Sarioglan **CT**

Department of Mathematics, Hacettepe University, Ankara, Turkey

In this talk, we focus on the freeness of generalized spline modules over cycles on bivariate polynomial rings. We show the freeness of generalized spline modules on graphs with no interior edges over $k[x, y]$ such as cycles or a disjoint union of cycles with free edges. We also consider graphs that can be decomposed into disjoint cycles without changing the isomorphism class of the syzygy modules. We use this decomposition to extend our results to other type of graphs and the multivariate polynomial rings. This is a joint work with Prof. Dr. Selma Altinok Bhupal.

Keywords: Generalized splines, graphs, polynomial rings, projective dimension, syzygy.

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When Enumeration Becomes an Algebraic Concern

Firdous Ahmad Mala[1,2] **CT**

[1]Chandigarh University, Mohali, India
[2]Government Degree College Sopore, Jammu and Kashmir, India

Abstract algebra is a very old branch of mathematics. It is the study of algebraic structures and their interactions. Symmetry could be considered a more relevant concern to algebra, yet it should not take one too much to gauge the fact that the study of abstract algebra is also flooded with numerous counting problems. Though enumeration is not central to the study of abstract algebra, yet the situations in abstract algebra where counting seems a more important concern than any other are not scarce. In this paper, we discuss several enumeration problems that arise in several problems primarily sprouting in abstract algebra. We review and summarize some of the most beautiful observations and theorems made in this direction. In particular, we discuss and expound upon the counting problems associated with the study of some special algebraic structures including associative binary operations, monoids, groups, rings, lattices, vector spaces, linear transformations and fields.

Keywords: Counting, enumeration, fields, groups, rings.

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In Memoriam Cem Tezer

Fatma Muazzez Simsir **CT**

Department of Mathematics, Selcuk University, Konya, Turkey

Cem Tezer was a fastidious, meticulous, highly idiosyncratic and versatile scientist. Without him Turkish community of mathematics would be incomplete. Our sense of gratitude for his work in various areas of mathematics, history of sciences, literature, music and his encouragement to do mathematics for only its beauty was hardly unique and even unusual. After he died on 27 February 2020, while working actively at Middle East Technical University, the number of colleagues and former students described the ways in which their studies and indeed their view towards mathematics had been transformed by having known him might have surprised only those who had never met him. In this talk not only, his contributions to mathematics will be classified and summarized but also his unique and distinguished personality as a mathematician will be emphasized.

Keywords: Dynamical systems and ergodic theory, general topology, geometry, global analysis, history of mathematics.

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[3] METU Mathematics Department. `https://math.metu.edu.tr/cem-tezer`

New Type of Metrics Deformations and Applications to Biharmonic Maps

Ahmed Mohammed Cherif[1], ***Bouchra Merdji***[1] **CT**

[1]Faculty of Exact Sciences, Mascara University, Algeria

We construct biharmonic non-harmonic maps between Riemannian manifolds (M, g) and (N, h) by first making the ansatz that $\varphi : (M, g) \to (N, h)$ be a harmonic map and then deforming the metric on N by $\tilde{h} = h - df \bigotimes df$ to render φ biharmonic, where f is a smooth function on N satisfying some conditions. We construct a new example of biharmonic non-harmonic map.

Keywords: Biharmonic maps, harmonic maps, Riemannian geometry.

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Some Results for Constant H_r Hypersurfaces Embedded in Pseudo-Riemannian Manifolds

Mohammed Abdelmalek **CT**

Department of Mathematics, Aboubekr Belkaid univesrity, Tlemcen, Algeria

In this work, we study the hypersurfaces of constant higher order mean curvature with spherical boundary embedded in a pseudo-Riemannian manifold. We prove firstly that under some conditions, the hypersurface M is transverse to an hypersurface P that contain its boundary. We also give a new proof of the Aleksandrov spherical caps conjecture and classify hypersurfaces of constant higher order mean curvatures embedded in a pseudo-Riemannian manifold.

Keywords: Higher order mean curvature, spherical caps, Newton transformations, elementary symmetric functions, Pseudo-Riemannian manifolds.

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Curvature Equality and Inequality for Sequential Warped Product Submanifolds

Anuj Kumar[1], ***Anil Sharma***[2] **CT**

[1]Srinivasa Ramanujan Department of Mathematics, Central University of Himachal Pradesh, India
[2]Department of Mathematics, Chandigarh University, Mohali, Punjab, India.

Recently in [8], Sahin presented the existence of sequential warped product submanifolds and derived geometric inequalities for such submanifolds in Kaehler manifolds. Motivated to this, in this article we investigate the geometry of sequential warped product submanifolds by developing curvature equality and inequality for submanifolds immersed in a nearly Kaehler manifold and generalized complex space form respectively. We first obtain equality involving mean curvature vector and second fundamental form for sequential warped product submanifold in a nearly Kaehler manifold and then presented the Lawson-Simons [6] type inequality in generalized complex space form that extends the result derived by Sahin in [8] for complex space form. We also examine the corresponding equality and special cases.

Keywords: Curvature inequalities, complex space forms, Riemannian manifolds, submanifolds, warped products.

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Real Structures in Singular Models of K3-surfaces

Cisem Gunes Aktas **CT**

Faculty of Engineering Sciences, Abdullah Gul University, Kayseri, Turkey

It is a wide open problem what kind of singularities a projective surface or a curve of a given degree can have. In general, this problem seems hopeless. However, in the case of K3-surfaces, the equisingular deformation classification of surfaces with any given polarization becomes a mere computation. In this talk, we will discuss projective models of K3-surfaces of different polarizations together with the deformation classification problems (see, [1] and [5]). Typically, over C, one deals with singular models, whereas, over R, with very few exceptions (see, e.g. [3] and [6]), one usually cofine oneself to the smooth ones. Certainly one could have considered singular real models up to equisingular equivariant deformations, but the results in lists would be huge[4]. In this research, we make an attempt to bridge this gap from a slightly different perspective, namely, we develop an algorithm detecting real representatives in equisingular strata of complex singular models of K3-surfaces. Then, we apply this algorithm to spatial quartics and find two new examples of real strata without real representatives. As a by-product, we also give a new proof for the only previously example of this kind in the space of plane sextics [1].

Keywords: Equisingular, polarization, projective models, quartics, real models.

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Harmonic Maps Between Berger-Type Deformed Sasaki Metric

El hendi Hichem **CT**

Department of Mathematics and Informatics, University Tahri Mohamed, Bechar, Algeria

Let $\psi : (M_{2k}, \phi, g) \to (N_{2k'}, \phi', h)$ be a smooth map between almost anti-paraHermitian manifolds. The map ψ induces the tangent map $\Psi : (TM, g^{BS}) \to (TN, h^{BS})$. In this paper, we deal with the harmonicity of a tangent map Ψ in the case where the tangent bundles TM, TN are endowed with the Berger type deformed Sasaki metric g^{BS}, h^{BS}.

Keywords: Horizontal lift, vertical lift, tangent bundles, horizontal Sasaki gradient metric, curvature tensor.

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Singular Minimal Surfaces and Perfect Domes in Architecture

Rafael López **CT**

Department of Geometry and Topology, University of Granada. Granada, Spain

If the catenary is the plane curve with the lowest center of gravity, a singular minimal surface is the corresponding analogue in the two dimensional case. So, the shape of this surface is obtained by a hanging flexible, inelastic material with prescribed area and boundary curve. In consequence, it is, after being turned upside down, a model for domes under the simple hypothesis of compression. In the first part of the talk, we show a novel design for a roof which has the property that its shape, although natural, is modeled by a surface of revolution whose axis of rotation is horizontal. In the second part, we investigate if catenary rotation surfaces and paraboloids are far from the ideal model, and in consequence, if both surfaces are useful candidates as a design model for the construction of roofs and domes.

Keywords: Singular minimal surface, center of gravity, catenary rotation surface.

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Approximation by Sampling Kantorovich Series in Weighted Space of Functions

Tuncer Acar[1], ***Osman Alagoz***[2], ***Ali Aral***[3], ***Danilo Costarelli***[4], ***Metin Turgay***[1], ***Gianluca Vinti***[4] **CT**

[1]Department of Mathematics, Selcuk University, Konya, Turkey
[2]Department of Mathematics, Bilecik Seyh Edebali University, Bilecik, Turkey
[3]Department of Mathematics, Kirikkale University, Kirikkale, Turkey
[4]Department of Mathematics and Computer Science, University of Perugia, Perugia, Italy

This talk deals with convergence of so-called generalized sampling Kantorovich operators for functions belonging to weighted spaces of continuous functions. This consideration allows us to extend convergence results without uniform continuity of target functions. The first and second parts are devoted to fundamentals of generalized sampling and sampling Kantorovich series, some auxiliary results as well. The third part is devoted to well-definiteness of the operators between weighted spaces of functions, and pointwise and uniform convergence. In the fourth part, we present rate of convergence of the family of operators in terms of weighted modulus of continuity. Also, a rate of convergence result for difference of generalized sampling and sampling Kantorovich operators is also obtained. The final part contains a pointwise convergence result in quantitative form by means of Voronovskaja theorem.

Keywords: Asymptotic formula, difference of operators, generalized sampling series, Kantorovich operators, pointwise convergence, weighted approximation.

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An Investigation on the Vertices of $F_{u,N}$ Obtained from Lucas Numbers in Hyperbolic Geometry

Ibrahim Gokcan[1], ***Ali Hikmet Deger***[2] **CT**

[1]Artvin Coruh University, Faculty of Science and Arts, Artvin, Turkey
[2]Department of Mathematics, Karadeniz Technical University, Trabzon, Turkey

In this study, Graph theory was studied on Hyperbolic geometry, which is one of the non-Euclidean geometries. [5] developed new concepts on suborbital graph motion. These concepts found an important place in applications for finite groups in studies of [4] and [6]. [3] investigated vertices of suborbital graph $F_{u,N}$ obtained with an element of Modular group Γ. [1] and [2] examined that each vertex in the suborbital graph $F_{u,N}$ has a continued fraction structure. [2] obtained the value of any vertex on a minimal length path from the type of Fibonacci numbers. In this study, the vertices of the suborbital graph $F_{u,N}$ were found as Lucas numbers using the identity $F_n \cong \frac{a_n}{\sqrt{5}}$. Identities related to Fibonacci and Lucas numbers were reached with the help of identity $p_n = (-1)^n F_{2n} \cong (-1)^n \frac{a_n}{\sqrt{5}} L_n$ where n^{th} numerator of the suborbital graph is p_n.

Keywords: Continued fraction, graph theory, identities, modular group, suborbital graph.

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A Result for F-Contraction on Orthogonal Metric Space

Aybala Sevde Ozkapu[1], ***Ozlem Acar***[1] **CT**

[1]Department of Mathematics, Selcuk University, Konya, Turkey

Fixed point problems is parts of the famous and traditional theories in mathematics and have a broad set of applications. The beginning of the metric fixed point problem is the famous Banach Contraction Principle given by Banach [3] in 1922. In this problem, contraction is one of the main tools to prove the existence and uniqueness of a fixed point. Banach's contraction principle which gives an answer to the existence and uniqueness of a solution of an operator equation $Tx = x$, is the most widely used fixed point theory in all of analysis. Afterwards, the crucial role of the principle in existence and uniqueness problems arising in mathematics has been realized which fact directed the researchers to extend and generalize the principle in many ways. In 2017, Gordji et.al. [2] defined the concept of an orthogonal set and gave an extension of the Banach Contraction Principle in orthogonal metric spaces and also they gave the application of this results for the existence and uniqueness of the solution of a first-order ordinary differential equation, while the Banach contraction mapping principle cannot be applied in this situation. On the other hand, K. Sawangsup et.al. [4] gave a generalization of Wardowski's theory [5] by defining the concept of F-contraction mapping in orthogonal metric spaces in 2020.

In this talk, we consider rational type F-contraction on orthogonal metric space and establish the existence of fixed point of such mapping. At the end, we give an example.

Keywords: Differential equation, fixed point, F-contraction, orthogonal metric space, rational type orthogonal F-contraction.

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Fixed Point Results for Almost F-Type Contraction

Esref Erdogan[1], ***Ozlem Acar***[1] **CT**

[1]Department of Mathematics, Selcuk University, Konya, Turkey

Fixed point theory on a metric space is started by Banach in 1922. Banach's Contraction Principle says that, whenever (X, d) is complete, then any contraction selfmap on X has a unique fixed point. A number of efforts have been made to enrich and generalize Banach Contraction Principle because it plays a fundamental role in metrical fixed point theory. Recently, for the first time, Gordji et. al. expand the literature on metric space by introduced the concept of orthogonality, obtained the fixed point result. There are several uses for this novel idea of an orthogonal set, as well as numerous forms of orthogonality. In their work, they made a generalization of the Banach fixed-point theory in this interestingly structure, and also applied their results to demonstrate the existence of a solution to the ordinary differential equation. In this talk, we describe a new Berinde-type contraction by combining the almost contraction with the F-contraction and we consider almost F-type contraction in the orthogonal metric space and detect the existence of a fixed point of such a mapping.

Keywords: Fixed point, F-contraction, orthogonal metric space.

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$3-$dimensional C_{12} Manifolds

Beldjilali Gherici **CT**

Laboratory of Quantum Physics and Mathematical Modeling (LPQ3M) University of Mascara, Mascara, Algeria

In 1960s and 1970s, the almost contact manifolds were studied as an odd dimensional counterpart of almost complex manifolds, the warped product was used to make examples of almost contact manifolds.

There are different classifications of almost contact structures, in [3], D. Chinea and C. Gonzalez have defined 12 classes of almost contact metric manifolds. According to [3], Theorem 5.1, in dimension 3, these manifolds are reduced to five classes: $|C|$ class of cosymplectic manifolds, C_5 class of β-Kenmotsu manifolds, C_6class of α-Sasakian manifolds, C_9-manifolds and C_{12}-manifolds.

So far, there are no detailed studies on the last two classes, as is the case for the first three.

This presentation is devoted to 3-dimensional C_{12}-manifolds which are never normal. Firstly, we study their fundamental properties and we give concrete examples. Secondly, we construct some relations between class C_{12} and other classes as C_6 and $C_2 \bigoplus C_9$ or $|C|$. On the other hand, we show there is a one-to-one correspondence between $C_{12}$and Kählerian manifolds. As an application, we study such structures on three dimensional Lie groups.

Keywords: Almost contact metric manifold, almost complex manifolds, warped product, Lie algebra.

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On the Geometry of Lift Metrics and Lift Connections on the Tangent Bundle

Esmaeil Peyghan[1], ***Davood Seifipour***[2], ***Adara M. Blaga***[3] **CT**

[1]Department of Mathematics, Arak University, Arak, Iran
[2]Department of Mathematics, Islamic Azad University, Abadan, Iran
[3]Department of Mathematics, West University of Timisoara, Romania

We study lift metrics and lift connections on the tangent bundle TM of a Riemannian manifold (M, g). We also investigate the statistical and Codazzi couples of TM and their consequences on the geometry of M. Finally, we prove a result on 1-Stein and Osserman structures on TM, whenever TM is equipped with a complete (respectively, horizontal) lift connection.

Keywords: Codazzi manifold, statistical manifold.

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Stability Analysis and Comparative Study on Different Eco-epidemiological Models: Stage Structure for Prey and Predator Concerning Impulsive Control

Sayooj Aby Jose[1], ***R.Raja***[2] **CT**

[1]Department of Mathematics, Alagappa University, Karaikudi, India.
[2]Ramanujan Centre for Higher Mathematics, Alagappa University, Karaikudi, India.

The asymptotic behaviour of four integrated pest management models with stage structure is analysed in this paper. Stage structuring is suggested because almost all pests pass through two stages in their lives, namely immature larvae and mature adults. It is believed that immature susceptible pests and exposed pests are targeted by a natural enemy and infected pests that make them exposed are contacted by susceptible pests (immature and mature). After fixed times, infected pests and natural enemies are infused impulsively. And we have also integrated the effects of non-target pests here. The stability analysis of the periodic pest extinction solution is obtained by the foquet theory, the method of small amplitude perturbation, and the comparison theorem. The results obtained provide some valid theoretical results for successful management of pests and also established the threshold time limit for the impulsive release of various controls. A comparative analysis with computational simulation offers a good idea of each model and our primary outcomes.

Keywords: Eco-epidemiological models, integrated pest management, impulsive effect, stability analysis, stage structure, target and non-target pest, Floquet theory.

A Kinematic Model of the Rytov's Law in the Optical Fiber via Split Quaternions: Application to Electromagnetic Theory

Zehra Ozdemir **CT**

Department of Mathematics, Amasya University, Amasya, Turkey

This paper aims to investigate the homothetic motion of the polarization plane traveling along the linearly polarized light wave ((LPL)-wave) in the optical fiber on the condition that the angle between the polarization vector **E** and the Frenet vector **t** (resp. **n** and **b**) is constant in 3D semi-Riemannian manifolds. Moreover, we present the relation of the homothetic motion and the Fermi-Walker parallel transportation law in 3D semi-Riemannian manifolds. The main technique for investigation of the homothetic motion is to use split quaternions. The parametric representations of the related Rytov curves are obtained through the split quaternion product and one-parameter homothetic motion. Then, we give some theorems and corollaries which show the relationship between the Rytov curves and the split quaternions in 3D semi-Riemannian manifolds. Moreover, using the variational vector field, we obtain the magnetic curves (EM–curves) associated with the polarization vector **E** obtained by the homothetic motion in the optical fiber. Furthermore, we illustrate some examples related to the theoretical results.

Keywords: Optical fibers, quaternion algebras, applications to physics, electromagnetic theory, variational methods, semi-Riemannian spaces, Berry phases.

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A Differential Inclusion of Second-Order and Application to Control

Soumia Saïdi **CT**

Department of Mathematics, Mohammed Seddik Ben Yahia University, Jijel, Algeria

Differential inclusions driven by time and state dependent maximal monotone operators have been considered recently in e.g. [3], [6]. Among their main applications are state-dependent sweeping processes and Lur'e dynamical systems, see [3].

Actually, there is an interest in differential inclusions of second-order governed by time and state dependent maximal monotone operators (and related variants). These operators vary in the sense of the pseudo-distance introduced in [7]. We refer to e.g. [1], [2], [4], [5], for the study of such systems with some of their applications to control theory and to fractional boundary problems. Our main goal is to provide a new investigation in this direction.

Keywords: Differential inclusion, maximal monotone operator, optimal control, pseudo-distance, second-order.

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N-dimensional Lattice Path Enumeration

Alper Vural[1], ***Cemil Karacam***[1], ***Mehmet Akif Karaca***[1] **CT**

[1]Department of Mathematics, Yildiz Technical University, Istanbul, Turkey

Let $V_n := \{(h_1, h_2, \ldots, h_n) : h_i \in \{0, 1\}, 1 \leq i \leq n\} - \{(0)\}$ be a set of integer vectors. We enumerate lattice paths that only uses vectors in V_n We find a formula that gives the number of paths from origin to any point in $n-$dimensional space. Then, we find the recursive relation between these numbers. We notice that a special case of this problem corresponds to Fubini numbers, which count the number of weak orderings of a set consisting of n elements. These are the numbers of arrangements of n competitors. The answer to this problem is not just $n!$ as competitors can have equal points. Lastly, we develop an algorithm that can be used to find the number of paths between any two points that do not stop at the lattice points in a given set of points, R. The algorithm we found does not apply only to V_n, it applies for any set of vectors.

Keywords: Binary paths, combinatorial function, diagonal steps, forbidden paths, Fubini number.

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Correlation Coefficients of Interval-Valued Fermatean Hesitant Fuzzy Sets with Their Application

Ibrahim Demir **CT**

Department of Mathematics, Yildiz Technical University, Istanbul, Turkey

An interval-valued fermatean hesitant fuzzy set is a combination of interval-valued fermatean fuzzy sets with fermatean hesitant fuzzy elements in the form of interval values. Therefore, interval-valued fermatean hesitant fuzzy sets are extensions of fermatean hesitant fuzzy sets, which are powerful tools to represent more complicated, uncertain, and vague information. This study aims to focus on four kinds of correlation coefficients for fermatean hesitant fuzzy sets and extend them to correlation coefficients and weighted correlation coefficients for interval-valued fermatean hesitant fuzzy sets. The least common multiple expansion method was developed to solve the problem of different cardinality of interval-valued fermatean hesitant fuzzy elements. The score functions and accuracy functions of interval-valued fermatean fuzzy elements to rank all the interval-valued fermatean fuzzy sets in an interval-valued fermatean hesitant fuzzy element have been proposed. At last, the numerical example shows the feasibility and applicability of the proposed methods in multiple criteria decision-making.

Keywords: Correlation, correlation coefficient, fermatean hesitant fuzzy set, informational energy, interval-valued fermatean hesitant fuzzy set.

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Accretive Surface Growth by Using Bézier Curve Fitting

Gul Tug[1], ***Selcuk Han Aydin***[1] **CT**

[1]Department of Mathematics, Karadeniz Technical University, Trabzon, Turkey

In this study, the fundamentals of the accretive surface growth process of some previously considered crusted creatures are analyzed. Also, several studies has been made on this concept. However, the curves in nature may be more complicated rather than a circle. Therefore, we described the accretive growth of the surfaces by using the elastic curves as the initial curves which are generators. However, the method is slightly complex in the case of elastic curves since it is necessary to solve a system of non-linear PDE which is a difficult task and may be obtained numerically. Therefore, the curves are approximated by using the Bézier curves that provide polynomial approximation for the initial curves and make possible to illustrate the corresponding accretive surfaces.

Keywords: Cubic Bézier curve, Darboux matrix, elastic curves, gowth velocity, PDE.

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Closed Space-like Ruled Surfaces thanks to Rotation Minimizing Frames

Ozgur Keskin[1], ***Yusuf Yayli***[1] **CT**

[1]Department of Mathematics, Ankara University, Ankara, Turkey

In this study, initially, integral invariants that are the pitch and the angle pitch of any $(k+1)$-dimensional closed space-like ruled surface are computed, considering any Rotation minimizing frame (RMF) in five-dimensional Minkowski space. In the first case, any $(k+1)$-dimensional closed space-like ruled surface is created according to the RMF that is obtained by taking the timelike vector field of the E_3 vector field of the Frenet vector fields $\{T, E_1, E_2, E_3, E_4\}$ of this space-like curve, and the orthogonal trajectory of this surface, the angle pitch, and the pitch are calculated. In the second case, $(k+1)$-dimensional closed space-like ruled surface is created as regards the RMF that is obtained by taking the timelike vector field of the E_1 vector field of the Frenet vector field $\{T, E_1, E_2, E_3, E_4\}$ of this space-like curve and the orthogonal trajectory of this surface, the angle pitch, and the pitch are figured out. Finally, all the obtained states are generalized to the n-dimensional Minkowski space, and all cases are examined as two states. Next, the calculations are shown in an example. Our aim in this study is to calculate the angle pitch and the pitch of closed ruled surfaces in high dimensional Minkowski spaces with the help of RMF. In the light of such information, it has been manifested the integral invariants can be computed more smoothly by means of the RMF.

Keywords: Angle of pitch, closed space-like ruled surface, Minkowski space, pitch, ruled surface, rotation minimizing frame (RMF).

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Closedness, Sobriety, Connectedness, and Compacteness in the Category of Constant Limit Spaces

Tesnim Meryem Baran[1], ***Ayhan Erciyes***[2] **CT**

[1]Ministry of National Education, Kayseri, Turkey
[2]Department of Mathematics, Aksaray University, Aksaray, Turkey

Lim category of limit spaces and continuous maps, is cartesian closed. The notions of local T_1, closedness, and closure operators are intensively used to define topological concepts such as regular and normal objects, compactness, and connectedness in abstract categories. The space X is called sober if every irreducible closed subset has a unique generic point. The sober spaces, which are one of the most important classes in the theory of non-Hausdorff topological spaces. They presented sober spaces in some topological categories and examined the relationship among T_0 spaces, irreducible spaces and sober spaces. Baran and Kula introduced the notions of connectedness and strong connectedness in set based topological categories. A compactness notion in categories of convergence spaces which depends on a closure operator given in 1987 by Dikranjan and Giuli. In 1997, Baran proved that some well-known results, in general topology, that are related to compactness. We characterize each of $T_i, i = 0, 1, 2$ closed quasi-sober, sober, irreducible, connected, and compact constant limit spaces. Furthermore, we prove that each of these subcategories consisting of T_i constant limit spaces for $i = 0, 1, 2$ and preT_2 are epireflective. In this way we investigate the relationship among these subcategories.

Keywords: Closure operator, sobriety, compacteness, connectedness, constant limit spaces.

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Stability Analysis of a Modified Holling-Tanner Model with Delay

Ali Demirci[1], ***Cihangir Ozemir***[1], ***Semih Sakarya***[1] **CT**

[1]Department of Mathematics, Faculty of Science and Letters, Istanbul Technical University, Istanbul, Turkey

Among many successful models such as the Lotka-Volterra system [1-2], Holling-Tanner model is a very effective one to show the true dynamics of the prey-predator interactions [3-4]. The main goal of this study is to introduce a new dynamic model in order to achieve a better understanding of the biological relations between prey and predators through a new dynamical system that includes a delay effect. The work here focuses on an interaction mechanism between two preys and one predator species and the suggested new model is proposed upon some modifications on the classical Holling-Tanner Type II model [4]. By analyzing the stability of the modified model, the dynamic response of this system is investigated and stability conditions at the equilibrium points are presented for both the non-delayed and delayed model. Furthermore, the specific analysis for a Hopf bifurcation is performed. With the numerical simulations at the end of the work, the results are supported numerically. The model that we introduced in this study can be modified for more than three species but with different interactions among them. This prospective development of the model will be topic of future studies.

Keywords: Prey-predator models, Hopf bifurcation, population dynamics, coexistence state, equilibrium.

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New Computation Method of Minimum Dwell Time for the Schur Stability of Discrete-Time Linear Systems

Gulnur Celik Kizilkan[1], ***Ahmet Duman***[1] **CT**

[1]Department of Mathematics-Computer, Necmettin Erbakan University, Konya, Turkey

Switched systems are continuous or discrete dynamic systems consisting of a finite family of systems and a rule that provides the transitions between these subsystems [1-2]. Switched systems are encountered in mathematical physics such as the mechanical systems, automotive industry, communication networks, aircraft and many other fields [1-3]. For this reason, it has been one of the research topics that has attracted attention recently. Naturally, as in all dynamic systems, it is important to examine the stability of the switched systems. In a switched system, even if all subsystems are stable, the system may be unstable for some switching signals [2,4]. In this study, we have considered the following discrete- time switched system:

$$x(t+1) = A_{\sigma(A)}x(t), t \in N \tag{0.1}$$

where $x(t) \in C^N, A_p \in C^{N\times N}$ are constant matrices, $p \in P, P$ is index set and $\sigma : N \to P$ is switching signal. We have obtained a dwell and average dwell time method which make the switched system (0.1) Schur stable. The method we obtained is based on the Schur stability parameter $\omega(A)$ of the system (0.1). For $\omega(A)$, see [5]. The dwell time methods in the literature are generally based on eigenvalues [6-8]. Although these eigenvalue-based methods are generally practical, they do not work well for non-symmetric matrices. Because the eigenvalue problem is an ill-posed problem for non-symmetric matrices [9-10]. As far as we know, “dwell” and “average dwell time” have not been studied depending on the $\omega(A)$ parameter in the literature. Therefore, the results obtained in this study are new and original.

Keywords: Average dwell time, Dwell time, Schur stability, stability parameter, switched systems.

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Set-valued Minimax Fractional Programming Problems via Second-order Contingent Epiderivative

Koushik Das **CT**

Department of Mathematics, Taki Government College, Taki, West Bengal, India

In optimization theory, many authors are interested to study minimax fractional programming problems. This type of problems arise in a large number of applications in different fields of mathematics, economics and engineering. Minimax fractional programming problems deal with extremal problems that involve constraints in a finite dimensional space, where the objective map and constraint are vector-valued. In this paper, generalized minimax fractional programming problems are studied for the case where the objective functions and the constraints involved in the problems are set-valued maps. We consider a set-valued minimax fractional programming problem.

$$\begin{aligned} &minimize_{x\in A} && max \bigcup_{y\in B} \frac{F(x,y)+(x^TD_1x)^{\frac{1}{2}}}{H(x,y)+(x^TD_2x)^{\frac{1}{2}}} \\ &subject \quad to && G(x)\cap(-R^P_+)\neq\emptyset \end{aligned} \tag{0.2}$$

where A is a nonempty subset of R^n, B is a nonempty compact subset of and R^m and $F, H : R^n \times R^m \to 2^R$ and $G : R^n \to 2^{R^P}$ are set-valued maps and D_1 and D_2 are $n \times n$ positive semidefinite matrices. In this paper, a set-valued minimax fractional programming problem (0.2) is introduced, where the objective function and functions attached to constraints are set-valued maps. The second-order sufficient KKT conditions are described for the problem (0.2) via second-order contingent epiderivative and second-order generalized cone convexity assumptions. Further, different types of duality models are formulated and a variety of duality relationships between the primal problem (0.2) and the corresponding second-order dual problems are established.

Keywords: Convex cone, cone convexity, duality, optimality conditions, set-valued map.

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High Degree B-Spline Algorithms for Numerical Solution of the Caudrey-Dodd-Gibbon Equation

Melis Zorsahin Gorgulu[1], ***Dursun Irk***[1] **CT**

[1]Department of Mathematics, Eskisehir Osmangazi University, Eskisehir, Turkey

In this study, the collocation finite element methods based on the octic (8th degree) and the nonic (9th degree) B-spline functions are used to get the numerical solutions of the Caudrey-Dodd-Gibbon equation (CDGE). The CDGE is given in [1] as

$$u_t + u_{xxxxx} + 30uu_{xxx} + 30u_xu_{xx} + 180u^2u_x = 0 \tag{0.3}$$

The Eq. (0.3) is a class of fifth order Korteweg-de Vries (KdV) equation which is used to model the well-known wave phenomena in the quantum mechanics and the nonlinear optics. The two simple numerical B-spline algorithms are proposed, taking into account the difficulties of getting the exact solution of the equation due to its non-linearity and having a fifth order derivative. For space discretization of the methods, the octic and the nonic B-spline functions are used as trial functions and for time discretization, the Crank-Nicolson method is used. What is considerable in this study is that the use of the high degree B-spline functions that have not been used before to solve numerically high order partial differential equations. The soliton wave solution of CDGE is simulated successfully to see the efficiency and the accuracy of the methods. The obtained error norm L_∞ values are considerably small for both of the methods.

Keywords: Collocation method, Crank-Nicolson method, nonic B-spline, octic B-Spline, soliton solution.

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Heat Transfer in Magneto Hydrodynamics Micropolar Jeffery Fluid Flow Through Porous Medium over an Exponentially Stretching Sheet with Thermal Radiation

Vandana Agarwal[1], ***Bhupander Singh***[2] **CT**

[1]Department of Computer Application, Raghunath Girl's (PG) College, Meerut, India
[2]Department of Mathematics, Meerut College, Meerut, India

In this paper numerical investigation of the problem of heat transfer in magneto hydrodynamics micro polar Jeffery fluid flow through porous medium over an exponentially stretching sheet with thermal radiation is examined using similarity solutions based on boundary layer assumptions. A parametric study for governing parameters such as magnetic field, porosity, Jeffery fluid, radiation, mixed convection, spin gradient viscosity and unsteadiness parameter etc. on the flow and heat transfer is conducted and their effects were discussed. A physical interpretation set of numerical results for the expressions like simulated velocity, mass concentration and micro rotation are plotted through graphs to show various trends. The numerical solutions for the micro rotation, simulated velocity and temperature profiles are computed, analyzed and discussed. Equations under the given problem with boundary conditions have been numerically solved and conclusions were drawn on the basis of these plotted graphs. The governing boundary layer equations are reduced into ordinary differential equations by employing the similarity transformation. The transformed equations are solved numerically using bvp4 package of MATLAB.

Keywords: Boundary layer, Deborah number, microrotation, Prandtl number, similarity transformation.

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A New Hypercube-like Graph

Burhan Selcuk[1], ***Ayse Nur Altuntas Tankul***[1] **CT**

[1] Department of Computer Engineering, Karabuk University, Karabuk, Turkey

The graph is used as a mathematical model in many branches of science. Graphs are a collaborative research field of mathematics and computer science. Hypercube graphs are used in computer architecture [1]. Many hypercube-like graphs have been described in the literature so far [2-6]. In this study, a new hypercube-like graph topology is introduced motivated by [7] and [8]. For this, two different definitions will be given. The first definition is derived from the union of a certain number of squares with a common side in two-dimension space. The second definition is derived from combining two squares or two connected square graphs obtained by a recursive process. The closed Hamiltonian paths of the new hypercube-like graphs are found with the help of gray code in [9] and, then these graphs are proved to be a Hamiltonian graph. A labeling algorithm finds the Hamiltonian paths for these graphs is obtained. The important graphical connectivity indexes of the new hypercube-like graphs are calculated as [10]. Finally, conclusions are given and a projection for future work is presented.

Keywords: Graph, gray code, hypercube, Hamiltonian path, graphical indices.

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Investigations on Singular Minimal Hypersurfaces in Lorentz Space

Ayla Erdur Kara[1], ***Muhittin Evren Aydin***[2], ***Mahmut Ergut***[1] **CT**

[1]Department of Mathematics, Tekirdag Namik Kemal University, Tekirdag, Turkey
[2]Department of Mathematics, Firat University, Elazig, Turkey

In this talk, we deal with the shape of a surface, called singular minimal surface, obtained by a hanging flexible, inelastic material with prescribed area and boundary curve. These surfaces minimize potential energy and generalize known minimal surfaces. Once inverted, such surfaces form a model for structures called domes. These types of structures are of great importance in architecture and therefore serve as a model for the application of geometry to architecture. They also physically form surface models with the lowest gravitational center. In addition to the results in the literature, we are looking for a solution to the problem "What would be the equivalent of such surfaces in Lorentz space?" and consider them to be timelike translation (hyper)surface. As a result, we obtain that these surfaces are generally either a (hyper)plane or a cylinder whose generatrix is α-Lorentzian catenary.

Keywords: α-catenary, cylinder, invariant surface, time-like surface, translation graphs.

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A Geometric Inequality for Warped Product Slant Submanifolds in Locally Metallic Product Space Forms

Mohd Aquib[1], ***Mohd Aslam***[2], ***Oguzhan Bahadir***[3] **CT**

[1]Department of Mathematics, Sri Venkateswara College, University of Delhi, New Delhi, India
[2]Department of Mathematics, Jamia Millia Islamia, New Delhi, India
[3] Department of Mathematics, Kahramamaras Sutcu Imam University, Kahramanmaras, Turkey

The concept of warped products is well-known in differential geometry and physics, particularly in general relativity. We can examine the energy, angles, and lengths of their second fundamental form using certain warped product manifolds. The best relativistic description of the Schwarzschid space time, which depicts the outer space around a big star or black hole, is provided as a warped product. In the present article, we construct a geometric inequality also know as Chen's inequality for square norm of the mean curvature vector and the warping function of warped product slant submanifolds in locally metallic product space forms and derive the condition under which equality to the inequality holds. Further, we discuss few applications in physics in terms of Hessian and Harmonic functions for the obtained result.

Keywords: Chen's inequality, harmonic function, Hessian function, θ-slant submanifolds.

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Each Ruled Surface can be Generated by a Real Quaternion

Selahattin Aslan[1], ***Yusuf Yayli***[1] **CT**

[1]Department of Mathematics, Ankara University, Ankara, Turkey

In this paper, we define concept of a moment curve of a ruled surface in three dimensional real vector space. The moment curve is just one curve for each ruled surface. We show that the ruled surface can be expressed by just one base curve instead of all base curves. Using the moment curve, we prove that each ruled surface can be generated by a real quaternion. The ruled surface is also expressed as a homothetic motion of director curve. Moreover, we show that this real quaternion represents 2 cylinders in four dimensional real vector space. Finally, we give examples of ruled surfaces using results obtained in this paper.

Keywords: Homothetic motions, moment curves, quaternion algebra, spatial rotations, two cylinders.

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F-Harmonic Maps and a Proper Homothetic Vector Field

Embarka Remli **CT**

Department of Mathematics, University of Mustapha Stambouli, Mascara, Algeria

Harmonic maps are solutions to a natural geometrical variational problem. This notion appear in various contexts: geodesic, harmonic functions, minimal surfaces...ect. We prove any f- harmonic map from a complete Riemannian manifold (M, g) to Riemannian manifold (N, h) is necessarily constant, with (N, h) admitting a proper homothetic vector field satisfying some conditions.

Keywords: Harmonic maps, f-harmonic maps, homothetic vector field.

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Ricci-like Solitons with Arbitrary Potential and Gradient Almost Ricci-like Solitons on Sasaki-like Almost Contact B-metric Manifolds

Mancho Manev **CT**

Department of Mathematics, University of Plovdiv Paisii Hilendarski, Plovdiv, Bulgaria

Ricci-like solitons with arbitrary potential are introduced and studied on Sasaki-like almost contact B-metric manifolds. A manifold of this type can be considered as an almost contact complex Riemannian manifold which complex cone is a holomorphic complex Riemannian manifold. The soliton under study is characterized and proved that its Ricci tensor is equal to the vertical component of both B-metrics multiplied by a constant. Thus, the scalar curvatures with respect to both B-metrics are equal and constant. In the 3-dimensional case, it is found that the special sectional curvatures with respect to the structure are constant. Gradient almost Ricci-like solitons on Sasaki-like almost contact B-metric manifolds have been proved to have constant soliton coefficients. Explicit examples are provided of Lie groups as manifolds of dimensions 3 and 5 equipped with the structures under study. First, a 5-dimensional example from a previous publication by the author is supported, which is in unison with the current theorems. After that, a 3-dimensional example with potential in general position is constructed. Finally, the latter example is shown to admit a gradient soliton of the type under consideration, supporting one of the theorems in the present article.

Keywords: Ricci soliton, η-Ricci soliton, Einstein-like manifold, η-Einstein manifold, almost contact B-metric structure, almost contact complex Riemannian manifold.

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Para-Ricci-Like Solitons with Arbitrary Potential on Para-Sasaki-Like Riemannian Π-Manifolds

Hristo Manev **CT**

Department of Medical Physics and Biophysics, Medical University, Plovdiv, Bulgaria

Para-Ricci-like solitons with arbitrary potential on para-Sasaki-like Riemannian Π-manifolds are introduced and studied. For the studied soliton is proved that its Ricci tensor is a constant multiple of the vertical component of both metrics. Thus, the corresponding scalar curvatures of the considered both metrics are equal and constant. Explicit examples of Lie groups as manifolds under study are presented.

Keywords: Almost paracontact, almost paracomplex, Einstein-like manifold, Riemannian manifolds.

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Application of Mixed Generalized Quasi-Einstein Spacetimes in General Relativity

Mohd Vasiulla **CT**

Department of Applied Sciences & Humanities, Jamia Millia Islamia (Central University) Jamia nagar, New Delhi, India.

In this paper, we study the mixed generalized quasi-Einstein $MG(QE)$ warped product spaces with non-positive scalar curvature. This note deals with investigating of some geometric and physical properties of $MG(QE)_n$ manifolds. Next, we study the general relativistic viscous fluid $MG(QE)_4$ spacetimes with some physical applications. Lastly, we show the existence of $MG(QE)_4$ spacetime by constructing a non-trivial example.

Keywords: Einstein manifold, mixed generalized quasi-Einstein manifold, warped product space, Einstein's field equation, energy-momentum tensor, general relativistic viscous fluid.

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Exploration of Pre-Open Sets in a Fuzzy Bitopological Space via Operation Approach

Satabdi Ray[1], ***Birojit Das***[1], ***Baby Bhattacharya***[1], ***Omer Kisi***[2] **CT**

[1]Department of Mathematics, NIT Agartala, India
[2]Department of Mathematics, Bartin University, Turkey

In this article, we propose the concept of an operation $(i,j)^*_\varphi$ on the family of all $(i,j)^*$-fuzzy-pre-open set in a given fuzzy bitopological space (X,τ_i,τ_j). Using the notion of $(i,j)^*_\varphi$ operation, we introduce the $(i,j)^*_\varphi$-fuzzy-pre-open and characterize it up to some extent. Also we, explore the notion of $(i,j)^*$-fuzzy pre-open set in the light of minimality. Finally, we study locally finiteness of a fuzzy bitopological space via $(i,j)^*_\varphi$-fuzzy-open set.

Keywords: $(i,j)^*_\varphi$-operation,$(i,j)^*_\varphi$-fuzzy-open set, $(i,j)^*$-fuzzy pre-open set, minimal $(i,j)^*$ fuzzy pre-open set, $(i,j)^*_\varphi$ -fuzzy locally finite pre-open set.

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New Methods of Construction for Ricci-Solitons

Adel Delloum[1], ***Beldjilali Gherici***[1] **CT**

[1]Department of Mathematics, University of Mascara, Mascara, Algeria

In general, the construction in geometry is a very wide field of research, the construction of structures on Riemannian manifolds will be specifically mentioned here, particularly the structure of Ricci-Soliton. The structure of Ricci-Soliton is recently developed in several works [see 2,5,7]. This research focus in the construction of important geometric tools such as special vector fields, the deformation of metrics, the product of manifolds and others. In 1982, Hamilton [see 10] introduced the notion of Ricci flow to find a canonical metric on a smooth manifold. It has become a powerful tool to study Riemannian manifolds, in particular those with positive curvature. This presentation focus on the geometry of transformations that is related to the (n-1)-dimensional distribution D of n-dimensional Riemannian manifold. The goal of this work is the construction of a Ricci-Soliton from a unit global vector field which will be the main tool for the deformation of the metric.

Keywords: Curvature, Einstein manifolds, deformation of metric, Riemannian manifolds, tensors.

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COPRAS Method Based on Entropy Measure under Complex Spherical Fuzzy Information

Ebru Aydogdu[1], ***Basak Aldemir***[2], ***Elif Guner***[1], ***Halis Aygun***[1] **CT**

[4]Department of Mathematics, Kocaeli University, Kocaeli, Turkey
[2]Department of Mathematics, Afyon Kocatepe University, Afyon, Turkey

Complex spherical fuzzy set (CSFS) is a generalization of spherical fuzzy set to express the two-dimensional ambiguous information in which the range of positive, neutral and negative degrees occurs in the complex plane with the unit disk. Considering the vital importance of the concept of complex spherical fuzzy sets which proficient huge attention in the research area of two- dimensional uncertain information, we aim to establish a novel methodology for multi-criteria group decision-making (MCGDM). For this goal, we first introduce a new entropy function under CSFS which measures the fuzziness degree associated with a CSFS. Also, we give a new formula based on entropy measure to evaluate the unknown criteria weights on this methodology. Then, we present an innovative Complex Proportional Assessment (COPRAS) based on the proposed entropy measure. Further, we give an illustrative study to show the feasibility and usefulness of the proposed method in the decision-making process.

Keywords: Complex spherical fuzzy sets, COPRAS method, entropy, MAGDM, MCGDM.

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A Note on Gradient Ricci-Harmonic Solitons

Fatma Karaca[1], ***Cihan Ozgur***[2] **CT**

[1]Department of Mathematics,Beykent University, Istanbul, Turkey
[2]Department of Mathematics, Izmir Democracy University, Izmir, Turkey

The Ricci-harmonic flow is one of the most useful tools in mathematics and physics, and it has been widely researched for recent years. Using the notion of Ricci-harmonic flow, Ricci- harmonic solitons were defined by Muller in [3]. Recently, the necessary and sufficient conditions for gradient Ricci harmonic soliton were obtained for the warped product manifold in [1]. Motivated by the above study, in the present study, we investigate gradient Ricci-harmonic solitons on doubly warped product manifolds. Firstly, we find the main relationships for doubly warped product manifolds to be a gradient Ricci-harmonic soliton. Secondly, we give various results for the potential function and the harmonic map which reaches maximum or minimum.Then, we obtain a gradient Ricci-harmonic soliton when the base and fiber are conformal to pseudo-Euclidean spaces which are invariant under the action of translation groups of codimension one. Finally, we give some physical applications for gradient Ricci-harmonic solitons on doubly warped product space-times.

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Keywords: Doubly warped product, gradient Ricci-harmonic soliton, Ricci-harmonic flow, space-time, warped product.

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Some Properties of Curvatures on the Total Manifold of a Riemannian Submersion

Semsi Eken Meric[1], ***Erol Kilic***[2], ***Erol Yasar***[1] **CT**

[1] Department of Mathematics, Mersin University, Mersin, Turkey
[2] Department of Mathematics, Inonu University, Malatya, Turkey

This paper deals with a Riemannian submersion between Riemannian manifolds such that total manifold admitting a Riemann soliton. Here, considering potential field of Riemann soliton, we study some properties of curvatures on the total space of Riemannian submersion and obtain some interesting results.

Keywords: Riemannian manifold, Riemann soliton, submersion.

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Spherical Fuzzy Sets: AHP-COPRAS Method based on Hamacher Aggregation Operator

Elif Guner[1], ***Basak Aldemir***[2], ***Ebru Aydogdu***[1], ***Halis Aygun***[1] **CT**

[1]Department of Mathematics, Kocaeli University, Kocaeli, Turkey
[2]Department of Mathematics, Afyon Kocatepe University, Afyon, Turkey

The aim of this study is to develop a model to solve the multi-criteria decision-making problems that selects the most suitable alternative by considering different criteria given their sub-criteria. For this aim, to use in the decision-making process we first define the Hamacher aggregation operator for the spherical fuzzy environment and study some algebraic properties of this operator. Then, we propose a novel method based on Hamacher aggregation operator for the spherical fuzzy environment consisting of two stages combined Complex Proportional Assessment (COPRAS) with the Analytic Hierarchy Process (AHP) method. In this process, the AHP method is used to select the criteria for the decision-making procedure and COPRAS is applied to solve the multi-criteria decision-making problems. Also, we give a numerical example to explain the proposed method clearly. Finally, we give a comparative study considering the existing methods in the spherical fuzzy environment to show the reliability and consistency of the new method.

Keywords: Analytic hierarchy process, complex proportional assessment, Hamacher operations, multi-criteria decision-making, spherical fuzzy set.

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From a Single Sasakian Manifold to a Family of Sasakian Manifolds

Zegga Kaddour **CT**

Department of Mathematics, Mascara University Mascara, Algeria

One of the goals of this paper is to explore other ways for constructing Sasakian manifolds. In the first part, we state and proof our first construction result [see Beldjilali and Belkhelfa (2016)] by another way and we give an example. In the second part, we construct Sasakian structures from Kählerian structures built above using a new technique and we construct a concrete example. Finally, we combine the two previous steps together to provide a way to create a 1-parameter family of Sasakian manifolds from just one Sasakian manifold directly.

Keywords: Product manifolds, Sasakian manifolds, Kählerian manifolds.

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On the Intersection Curve of Two Surfaces in E^n

Bedia Merih Ozcetin[1], ***Mustafa Duldul***[1] **CT**

[1]Department of Mathematics, Yildiz Technical University, Istanbul, Turkey

Surface-surface intersection is an important topic in CAGD. Since surfaces can be given by either its parametric or implicit equation, intersection problem of two surfaces occurs as parametric-parametric, parametric-implicit or implicit-implicit intersection. The parametric-parametric surface intersection problem has been studied in Euclidean n-space recently. However, other intersection problems have not been studied. In this study, we study the intersection problem of two surfaces in which at least one surface is given by its implicit equation in Euclidean n-space. To study such a problem, we first obtain the higher order derivatives of a curve lying on an implicit surface. Then, we obtain the curvatures of the transversal intersection curve of two implicit surfaces. Later, the intersection problem is studied for a parametric surface and an implicit surface. Finally, we present examples as applications of given methods.

Keywords: Curvatures, Frenet vectors, implicit curve, surface-surface intersection, transversal intersection.

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Geometries Related to the Critical Values of Calibrations

Ibrahim Unal **CT**

Department of Mathematics, Middle East Technical University, Ankara, Turkey

A calibration φ on a Riemannian manifold (M, g) is a closed differential p-form which satisfies the inequality

$$\varphi|_\xi \leq vol|_\xi$$

for any oriented p-dimensional subspace $\xi \in T_xM$ for any $x \in M$. Submanifolds corresponding to the absolute maximum value of a calibration, namely calibrated submanifolds, give us very rich examples of minimal submanifolds. Furthermore, it is showed that submanifolds corresponding to other non-zero critical values of a calibration may also be minimal under additional conditions. We provide some examples of these conditions and give examples of submanifolds corresponding to the critical values of a calibration other than the absolute maximum value. Especially, we focus on the example coming from the quaternionic calibration where the calibrated submanifolds are not interesting enough.

Keywords: Calibrated submanifold, complex Lagrangian, Grassmannian, minimal submanifold, φ-critical submanifold.

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A Novel Entropy Measure Based on Similarity Measure for Spherical Fuzzy Sets and Its Application on Multicriteria Decision Making Problems

Ali Aydogdu **CT**

Department of Basic Sciences, Turkish National Defence University, Izmir, Turkey

The concept of spherical fuzzy sets is one of the tool of fuzzy logic which overcome the uncertainty vagueness in reaching decisions in real life. Spherical fuzzy set (SFS), which is a generalization of Pythagorean fuzzy set (PFS) and neutrosophic set (NS), allow the decision- maker(s) to introduce their hesitancy about an attribute or alternative. Hence, Spherical fuzzy set (SFS) give a better representation possibility for expert's preferences, mathematically and applicatively. Also, entropy measure is a very important mathematical tool for measuring fuzziness degree and it has received great interest in the past decades as a significant quantitative measure of uncertain information. In this study, we propose a similarity measure between the spherical fuzzy sets. Then, we introduce a novel entropy measure for spherical fuzzy sets based on the proposed similarity measure. Also, we show that the new entropy measure provides required properties. Finally, we give a simple application in multi criteria decision making problem.

Keywords: Spherical fuzzy set, similarity, entropy, multi criteria decision making.

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R-Complex Finsler Spaces with An Exponential Metric

Ramdayal Singh Kushwaha[1], ***Renu Gill***[1]

[1]Department of Mathematics and Statistics, Banasthali Vidyapith, Banasthali, India

In this paper, we have defined the concept of the R-complex Finsler space with an exponential metric $F = \alpha e^{\frac{\beta}{\alpha}}$. For this metric, we have obtained the funda- mental metric tensor fields g_{ij} and $g_{i\bar{j}}$ together with their determinants and inverse tensor fields. Further, some properties of non-Hermitian R-complex Finsler spaces with this metric have been described.

Keywords: Complex Finsler space, R-complex Finsler space, fundamental metric tensors.

On Basicity Of The System of Eigenfunctions for One Discontinuous Operator Produced by a Spectral Problem in Banach Function Space

Fatih Sirin[1], ***Yusuf Zeren***[2] **CT**

[1]Department of Computer Programming, Istanbul Aydin University, Istanbul, Turkey
[2]Department of Mathematics, Yildiz Technical University, Istanbul, Turkey

In this study, the basicity properties of the eigenfunctions system of a second order differential operator containing spectral parameters under boundary conditions in Banach function spaces are investigated. Investigations are divided into two groups on whether spaces are rearrangement-invariant or not. With certain conditions placed on the Boyd indices of rearrangement-invariant Banach function spaces, the theorems about the basicity of the eigenfunctions system of the spectral problem in suitable separable subspaces of these spaces have been proved. Specifically, these theorems include Lebesgue, grand-Lebesgue, Orlics etc. Also, results regarding the basicity of the system of eigenfunctions in non-rearrangement invariant spaces of spaces with finite Boyd indices have been obtained. The discontinuity of the differential operator makes it possible to examine the basicity properties of the system of eigenfunctions in the direct sum space of spaces with finite Boyd indices.

Keywords: Rearrangement-invariant space, theory of close base, non-rearrangement invariant, Banach function spaces, Boyd indices.

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Hybrid CAS and Block Pulse Functions for Numerical Solution of Fractional Population Growth Model

Yahya Rahrovi[1], ***Yaghoub Mahmoudi***[1] **CT**

[1]Department of Mathematics, Tabriz Branch, Islamic Azad University, Tabriz, Iran

For a long time, fractional calculations have improved their position and efficiency in various fields of science such asmathematics, engineering and technology, biology, social sciences and sociology, etc. For example applications such as modeling of nonlinear earthquake oscillations, color vibrations, fluid dynamics, dynamic economics, control theory, signal processing and so on can be mentioned. In the present paper, a computational technique is proposed to solve the fractional population growth model. Otherwise, we solved the current model by using CAS wavelets and Block Pulse functions with collocation technique. For this purpose, the operational matrix of fractional integral was used for cosine and sine wavelets. The error estimation is provided. Finally, numerical solution of presented model and a comparison with two other methods are presented to show the validity and efficiency of the technique.

Keywords: Block pulse functions, wavelets, fractional differential equation, numerical approximation, operational matrix, Volterra integral equations.

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Weighted Minimal Translation Surfaces according to Semi-Symmetric Connections in Euclidean and Minkowskian 3-Space with Density

Mustafa Altin **CT**

Technical Vocational School, Bingol University, Bingol, Turkey

Translation surfaces in the Euclidean 3-dimensional and Minkowskian3-dimensional spaces are surfaces that can be written as the sum of two curves and they can be represented as graphs $z = f(x) + g(y)$, where f and g are smooth functions. In the present study, we study the type-I ($z = f(x) + g(y)$), type-II ($y = f(x) + g(z)$) and type-III ($x = f(y) + g(z)$) translation surfaces in the 3-dimensional Euclidean and Minkowskian spaces with density. In this context, we obtain the weighted mean curvatures of the three types of translation surfaces with respect to a kind of semi-symmetric metric and a kind of semi-symmetric non-metric connections in E^3 and E_1^3 with density e^{az}, $a \in R - 0$, separately. After obtaining the weighted mean curvatures with respect to a kind of semi-symmetric metric and a kind of semi-symmetric non-metric connections of these surfaces in these spaces with density e^{az}, we classify the weighted minimal translation.

Keywords: Density, semi-symmetric metric connection, semi-symmetric non-metric connection, translation surface, weighted mean curvature.

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Induced Structures on Generalized Geometry

Fernando Etayo[1], ***Pablo Gómez-Nicolás***[1], ***Rafael Santamaría***[2] **CT**

[1]Department of Mathematics, Statistics and Computing, University of Cantabria, Santander, Spain.
[2]Department of Mathematics, University of León, León, Spain.

The generalized tangent bundle $TM \bigoplus T^*M \to M$ was introduced by N. Hitchin [3] in 2003 and further developed by M. Gualtieri [2]. This vector bundle is also called big tangent bundle [6]. In this bundle, generalized almost complex structures have been defined as section endomorphisms of it. Almost complex and almost symplectic structures defined on the base manifold M induce objects. There are three canonical structures acting on the sections of $TM \bigoplus T^*M$: a metric called canonical pairing, a symplectic structure and an almost paracomplex structure, introduced by A. Wade [7]. Apart from its own relevance as a vector bundle, this construction can be used to obtain other geometric structures in other bundles (for example, a Norden structure on the cotangent bundle T^*M [5]). It is also useful in the development of other science fields, such as the geometric context of string theory [3]. In this presentation, we will observe how multiple generalized structures arise when the base manifold M is endowed with different geometric structures. In particular, we will show some examples studied in recent works by M. Gualtieri [2], A. Nannicini [5], C. Ida and A. Manea [4], and E. A. Fernández-Culma, Y. Godoy and M. Salvai [1], together with many other new examples. This presentation is based on a joint work with F. Etayo and R. Santamaría.e, almost paracomplex structure, almost product structure, almost symplectic structure, generalized tangent bundle.

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On a Type of Lightlike Submanifold of Poly-Norden Semi-Riemannian Manifolds

Tuba Acet[1], ***Selcen Yuksel Perktas***[2], ***Erol Kilic***[1] **CT**

[1]Department of Mathematics, Faculty of Science and Arts, Inonu University, Malatya, Turkey
[2]Department of Mathematics, Faculty of Science and Arts, Adiyaman University, Adiyaman, Turkey

In this paper, the lightlike geometry of poly-Norden semi-Riemannian manifolds have been studied by investigating the screen transversal Cauchy-Riemann lightlike submanifolds. We obtain integrability conditions for the distributions involved in the definition of such lightlike submanifolds. We present necessary and sufficient conditions for the distributions to define totally geodesic foliations and give an example.

Keywords: Bronze mean, CR-lightlike submanifold, distribution, golden ratio, totally geodesic foliation.

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Structure of F_qR-linear (Θ, Δ_Θ)-cyclic Codes

Shikha Patel[1], ***Om Prakash***[1] **CT**

[1]Department of Mathematics, Indian Institute of Technology Patna, Patna, India

Let F_q be the finite field of order $q = p^m$ where p is an odd prime and m is a positive integer. This work introduces F_qR-linear (Θ, Δ_Θ)-cyclic codes where $R = F_q + uF_q$ with $u^2 = 1$, that is, (Θ, Δ_Θ)-cyclic codes over F_qR. With the help of the decomposition method, we study the structural properties and determine the generator polynomials of F_qR-linear (Θ, Δ_Θ)-cyclic codes. Further, we define the Gray map over F_qR and find the Gray images of F_qR-linear (Θ, Δ_Θ)-cyclic codes over F_q. Finally, with the help of our established results, we have constructed many optimal codes corresponding to F_qR-linear (Θ, Δ_Θ)-cyclic codes.

Keywords: Finite field, gray map, linear codes, skew cyclic codes, skew polynomial rings.

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Multiplicity of Solutions for a Nonhomogeneous Elliptic Equations with Critical Exponent

Khadidja Sabri **CT**

Laboratory of Analysis and Control of Partial Differential Equations of Sidi Bel Abbes, University of Oran, Oran, Algeria.

This work deals with the existence and multiplicity of solutions to the following problem

$$\begin{cases} -\text{div}\left(\frac{|\nabla u|^{p-2}\nabla u}{|x|^{pa}}\right) - \mu\frac{|u|^{p-2}\nabla u}{|x|^{p(a+1)}} = f(x)\frac{|\nabla u|^{p_*-2}\nabla u}{|x|^{p_*b}} + \lambda g(x) \\ u \in W_a^{1,p} \end{cases} \tag{0.4}$$

where $1 < p < N, 0 \leq a < \frac{N-p}{p}, N \geq 3, a \leq b < a+1, 0 \leq \mu < \bar{\mu_a} := \left(\frac{N-p(a+1)}{p}\right)^p$ and λ is a positive parameter, $p_* = \frac{pN}{N-p(a+1-b)}$ is the critical Caffarelli-Kohn-Nirenberg exponent, $g \in (W_a^{1,p})'\backslash\{0\}$, (dual of $g \in (W_a^{1,p})$) is a continuous function such that $g(0) > 0$ and f is a bounded positive function on R^N. Here $W_a^{1,p} = W_a^{1,p}(R^N)$ denotes space obtained as the completion of the space $C_0^\infty(R^N)$ with the standard norm

$$||u|| = \left(\int_{R^N} \frac{|\nabla u|^p}{|x|^{pa}} - \mu\frac{u^p}{|x|(a+1)}\right)^{\frac{1}{p}}.$$

Problem (0.4) is related to the following well known Caffarelli-Kohn-Nirenberg inequality: there is a positive constant $C_{a,b}$ such that

$$\left(\int_{R^N} \frac{|u|^{p*}}{|x|^{p*b}}\right)^{\frac{1}{p^*}} \leq C_{a,b}\left(\int_{R^N} \frac{|\nabla u|^p}{|x|^{pa}}\right)^{\frac{1}{p}}$$

The main tool is variational methods, more precisely, by using the Ekeland's variational principle we can find the first critical point with negative level. From the Mountain Pass Theorem we also obtain a critical point whose level is positive.

Keywords: Critical exponent, Ekeland's variational principle, mountain pass lemma, Palais-Smale condition, variational methods.

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Solving Black-Scholes Equations for Barrier Option Pricing Using Adaptive Differential Evolution Neural Network Algorithm

Werry Febrianti[1], ***Kuntjoro Adji Sidarto***[1], ***Novriana Sumarti***[1] **CT**

[1]Department of Mathematics, Bandung Institute of Technology, West Java, Indonesia

Black-Scholes (BS) Equations is fundamental equations in mathematical finance especially in option pricing. This BS equations is stochastics partial differential equations that is not easy to solve numerically. In this paper, we propose a method to solve BS Partial Differential Equations that change the BS equations into an optimization problem. We use a metaheuristic optimization algorithm to solve that problem. In this paper, we use Adaptive Differential Evolution Neural Network (ADENN) algorithm to find the approximation solution for this BS problem. Therefore, we can find the approximation solution for BS Partial Differential equations using our ADENN algorithm. Then, we also find the approximation value of European option pricing and Barrier option pricing using our optimization method. The result of our approximation method fits well to the exact solutions.

Keywords: Adaptive differential evolution neural network, approximation solution, Black-Scholes, metaheuristic optimization, partial differential equations.

Comparison of Bezout Elimination Method and a New Method for Solving Nonlinear Systems of Equations of Second Order

Ercan Duzgun[1], ***Osman Kopmaz***[1] **CT**

[1]Mechanical Engineering Department, Bursa Uludag University, Bursa, Turkey

A new mathematical method is investigated for solving nonlinear system of equations with three unknown variables in three equations in a special format. The proposed method is based on finding the roots of a second order algebraic equations (polynomials) which have a symmetry between the unknowns. In this case the roots of any two of these equations are found in terms of one of these unknowns. Afterwards these roots are substituted in the remaining equation, which leads to a 32 degree polynomial after some symbolic equations, while using the Bezout Elimination method, it yields to a 16th degree polynomial by calculating symbolic determinants of 4x4 and 6x6 matrices which is comparatively difficult. The proposed method is easy to implement while still provides all roots. In the practice, the Stewart-Gough platforms used in the robotics/airplane simulators need the solutions of this kind of system of equations. The presented technique can be used for the forward kinematics problem to find possible manipulator assembly modes as well as any kind of nonlinear systems of equations in the same format as given here by using only simple mathematical operations that doesn't need special mathematical techniques.

Keywords: Bezout, nonlinear, polynomials, robotics, roots.

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Multiframelet and Multiframelet Set on Q_p

Debasis Haldar **CT**

Department of Mathematics, National Institute of Technology Rourkela, Odisha, India

Multiframelet is a generalization of wavelet theory as well as frame theory. We shall present our research work of various necessary conditions of p-adic multiframelet and multiframelet set. Multiframelet is a frame-like sequence generated by finitely many functions along with wavelet structure. We shall show various properties of multiframelet and multiframelet operator in $L^2(Q_p)$. Furthermore, multiframelet set on Q_p will be presented with examples. Multiwavelet sets are multiframelet set, but here we will give examples of multiframelet set which are not a multiwavelet set. Properties of semi-orthogonal multiframelets will be presented.

Keywords: Multiframelet, multiframelet set, p-adic, Parseval, semi-orthogonal.

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Positive Solution for Critical Elliptic System with Neumann Condition

Hamzaoui Yamina **CT**

Dynamic System and Applications Laboratory, Abou Bakr Belkaid University of Tlemcen, Tlemcen, Algeria

We consider the solvability of the Neumann problem for an elliptic system of two equations with weights involving two critical Sobolev exponents on a bounded domain in R^N. By using variational methods, we investigate the effect of the shape of the graph of the weight functions and the geometry of the boundary on the existence of solutions. The aim of this talk is to discuss the existence of solutions to the following nonlinear elliptic system

$$\begin{cases} -div(p(x)\nabla u) & = au + bv + \eta|u|^{2^*-2}u \text{ in } \Omega \\ -div(p(x)\nabla u) & = bu + cv + \eta|u|^{2^*-2}v \text{ in } \Omega \\ \frac{\partial u}{\partial n} & = \alpha Q(x)|u|^{\alpha-2}u|v|^{\beta} \text{ on } \partial\Omega \\ \frac{\partial u}{\partial n} & = \beta Q(x)|u|^{\alpha}|v|^{\beta-2}v \text{ on } \partial\Omega \end{cases}$$

where Ω is an open bounded domain in R^N with smooth boundary $\partial\Omega$, $N \geq 3$, n is the unit outward normal on $\partial\Omega$, p and Q are given positive and continuous weights defined on $\bar{\Omega}, \partial\Omega$ respectively with $p \in H^1(\Omega)$, a, b and c are real parameters, η is a nonnegative constant, $\alpha, \beta > 1$ such that $\alpha + \beta = 2_*$. Here $2_* = \frac{2(N-1)}{N-2}$ is the critical Sobolev exponent for the trace embedding $H^1(\Omega)$ into $L^{2*}(\partial\Omega)$ and $2* = \frac{2N}{N-2}$ denotes the critical Sobolev exponent for the embedding $H^1(\Omega)$ into $L^{2*}(\Omega)$. This kind of equations expresses the relationship between different domains as biological, physics and mathematics. For example the study of bacterial motion in terms of reaction diffusion leads to this type of problems. Let give some different conditions: $((H_\Omega) : \Omega \subset R^N, (N \geq 3)$ is a bounded domain such that $\partial\Omega$ satisfies a geometrical condition at x_0 and the mean curvature of the boundary at x_0 is positive. The functions p and Q satisfy the following conditions

$$\frac{p(x_0)}{(Q(x_0))^{N-2}} = min\frac{p(x)}{(Q(x))^{N-2}} \tag{0.5}$$

$$|p(x) - p(x_0)| = o|x - x_0| \tag{0.6}$$

$$|Q(x) - Q(x_0)| = o|x - x_0| \tag{0.7}$$

and

$$(A) : ac - b^2 > 0, a + c < 0 \text{ then } \mu_1 \leq \mu_2 < 0$$

These is the main results of this talk.
Theorem 1 Assume that (H_Ω) holds, p, Q satisfy $(0.1) - (0.3)$ and (A) is verified. Then our system admits a solution.

Keywords: Critical Sobolev exponents, Palais-Smale condition, concentration-compactness principle, Neumann problem.

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Four Classes of Perfect Nonlinear Functions

Dhananjay Kumar[1], ***Rajesh P. Singh***[2], ***Rishi Kumar Jha***[1] **CT**

[1]Department of Mathematics, National Institute of Technology Patna, Patna, Bihar, India
[2]Department of Mathematics, Central University of South Bihar, Gaya, Bihar, India

Let $q = p^n$ and p be an odd prime. Let F_q be a finite field of characteristic p. A function $f : F_q \to F_q$ is called perfect non-linear or planar, if for every non-zero $b \in F_q$ the discrete derivative $f(x+b) - f(x)$ of $f(x)$ is a bijection of F_q. Perfect non-linear functions have a wide range of applications in Combinatorics, Coding Theory, Finite Geometry, Cryptography, and Design Theory. A polynomial of the form $\sum_{i,j} a_{ij} x^{p^i+p^j}$ is called Dembowski-Ostrom polynomial (DO polynomials). DO polynomials were introduced by P. Dembowski and T.G. Ostrom in 1988, where they used perfect non-linear DO type polynomials to construct finite projective planes and affine planes in Finite Geometry. Only six inequivalent classes of perfect non-linear functions over a finite field F_{p^n} for arbitrary characteristic p and three inequivalent classes over a finite field of characteristic three are known so far. All 3s +1 known planar functions are of DO type with the only exception $x^{\frac{3^s+1}{2}}$ over a finite field of characteristic 3. In this article, we construct four classes of perfect nonlinear functions over finite fields $F_{p^{2n}}, F_{p^{3n}}$ and $F_{p^{4n}}$. All the proposed classes of the polynomials are of the DO type.

Keywords: Dembowski-Ostrom polynomial, finite geometry, planar functions, permutation polynomials, semifields.

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A Block Quaternion GMRES Method for Linear Matrix Equations

Sinem Simsek **CT**

Department of Mathematics, University of Kirklareli, Turkey

We consider the numerical solution of the linear quaternion matrix equation $AX = B$ for the unknown matrix X, where A, B are given nxn, nxs matrices over the skew-field of quaternions. Such quaternion equations arise in fields such as image and signal processing. Our primary concern is the large-scale setting when n is large so that direct solutions are not feasible. We introduce a block Krylov subspace method, indeed a block GMRES method, for the iterative solution of these quaternion matrix equations. The main challenge in the quaternion setting compared to usual block Krylov subspace methods over real or complex Euclidean spaces is that the multiplication of quaternion scalars is not commutative. Consequently, one has to employ a proper inner product defined over quaternion spaces, and the generation of an orthonormal basis for the Krylov space should be performed with care by taking noncommutativity features of quaternions into account. After adopting a proper inner product over quaternion spaces, we describe a block Arnoldi process to generate an orthonormal basis for the Krylov space $span\{R_0, AR_0, , AR\}$, where $R = B - AX_0$ and X_0 is an initial guess for the solution. Then the best solution of $AX = B$ in the least-squares sense is sought in the generated Krylov space. We explain how these least-squares problems over quaternion Krylov spaces can be solved efficiently by making use of Householder reflectors. It would not be possible to solve large quaternion matrix equations directly, and, to our knowledge, our approach is the first block Krylov subspace method proposed for their solution.

Keywords: Arnoldi process, householder reflector, inner product space, Krylov subspace methods, skew-field.

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Linear Skew Cyclic Codes over FpR

Tulay Yildirim Turan **CT**

Department of Mathematics, Karabuk University, Karabuk, Turkey

In this paper, we consider skew cyclic codes over the ring F_pR, where $R = F_p + vF_p$ for a prime p and $v^2 = v$. We investigated the algebraic structures of these codes in details. We described the Gray map of skew cyclic codes over the ring F_pR, also determine its generator polynomials. Further, we defined a nondegenerate inner product and provided a criterion to test for self-orthogonality. Finally, we established a natural connection with the DNA codes.

Keywords: Codes over rings, gray map, linear codes, mixed alphabets, skew polynomial rings.

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Statistical Variants of Analogs of Korovkin type Theorems in Banach Function Spaces

Yusuf Zeren[1], ***Migdad Ismailov***[2], ***Cemil Karacam***[1] **CT**

[1]Department of Mathematics, Yildiz Technical University, Istanbul, Turkey
[2]Institute of Mathematics and Mechanics of the NAS of Azerbaijan, Baku State University, Baku, Azerbaijan

This work deals with the analogs of statistical Korovkin theorems in Banach function spaces with non–standart growth. Morrey spaces, grand Lebesgue spaces and many non-standart spaces are not separable and this, in turn creates specific diffuculties in studying approximation theory for sequences of positive linear operator. For these spaces, a general subset to which the statistical Korovkin theorems can be applied has been determined. It was determined under which conditions statistical Korovkin theorems were valid. Under these conditions, statistical Korovkin theorems were expressed and proved in Banach function spaces. These results are obtained for Lebesgue space, grand Lebesgue spaces, Morrey-type spaces. Our study will determine the validity conditions of statistical Korovkin theorems in Banach function spaces and will help the studies for many linear operator sequences, especially Kantorovich polynomials in these spaces.

Keywords: Kantorovich polynomial, Korovkin theorems, non-standart spaces, statistical convergence, shift operator.

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Analysis of Boundary Layer Flow over a Moving Plate in a Porous Medium

Suman Sarkar[1], ***Bikash Sahoo***[1] **CT**

[1]National Institute of Technology, Rourkela, Odisha-769008, India

This paper deals with the existence and uniqueness of a third-order nonlinear differential equation over a semi-infinite domain $[0, \infty]$. This equation arises in the viscous fluid flow of a rarefied gas-free stream over a moving plate embedded in a porous medium. This work assumes that both the plate and free stream velocities are constants, and they are moving in the same direction. By choosing suitable similarity transformations, the mass and momentum boundary layer equations reduce to a self-similar nonlinear ordinary differential equation containing the following parameters: γ, the ratio of velocities, ω, permeability parameter, λ, slip parameter. It is seen that the solutions are concave or convex for different values of the parameters, and in some cases, the differences in proofs and results may be significant. Both the matters are discussed thoroughly. Further, the numerical results are obtained over the entire range of parameters and are compared with known results for some particular cases of this present work. The influence of the permeability and slip parameters on the velocity profiles and shear stress are addressed through a table, and some figures and possible discussions on the results are explained in detail. It is observed that the permeability parameter doesn't affect remarkably on the flow problem for the large value of the slip parameter.

Keywords: Existence and uniqueness, topological shooting argument, moving plate, partial slip, porous medium.

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Analytical methods in the Theory of Nonlinear PDEs: Two-dimensional Hirota System of Equations

Gaukhar Shaikhova[1], ***Bayan Kutum***[2], ***Ratbay Myrzakulov***[1] **CT**

[1]L.N. Gumilyov Eurasian National University, Nur-Sultan, Kazakhstan
[2]E.A. Buketov Karaganda State University, Karaganda, Kazakhstan

It is well known that nonlinear partial differential equations have attracted a lot of attention among researchers. This fascinating subject of nonlinear science has branched out in almost all areas of technology and science [1-3]. In this work, we focus on the construction of exact solutions for the two-dimensional Hirota system of equations, which is nonlinear partial differential equations [4]. We apply Darboux transformation [5-6] to obtain different kinds of solutions from a trivial seed. Additionally, we use the tanh method to obtain new exact solutions. The tanh method constructs traveling wave solutions in terms of hyperbolic tangent functions [7]. As follows, new families of solitary wave solutions and periodic solutions are obtained for the two-dimensional Hirota system of equations. Moreover, the two-dimensional Hirota system of equations can reduce to the two-dimensional nonlinear Schrodinger equation [8] and the two- dimensional complex modified Korteweg-de Vries equation [9]. The nonlinear Schrodinger equation is a universal model, that describes the evolution of nonlinear waves in media with cubic nonlinearities. The complex modified Korteweg-de Vries equation has been suggested as a model for the nonlinear evolution of plasma waves [10]. Thus, the obtained results can apply to several areas of physics.

Keywords: Darboux transformation, exact solutions, ODE, PDE, tanh method.

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An application of Graf Neural Networks in Diabetes Mellitus Protein Interactions

Alfi Y. Zakiyyah **CT**

Mathematics Department, Bina Nusantara University, Jakarta, Indonesia

A graph is a data structure that represents a set of objects and their relationships. Graphs can be used to represent and analyze big data such as search engine problems, social media, knowledge discovery, network systems, and others. Graph Neural Networks (GNNs) is a type of Neural Network that operates directly on a graph structure. The GNNs working system is the classification of points on a graph structure by Labbeling the node. Graph neural networks (GNNs) is a continuous study of developing the concept of neural networks. Artificial Neural Networks, abbreviated as artificial neural networks, is an information processing system that is appropriate for biological neural networks. In this research we analysis the application of GNNs in diabetes mellitus protein interactions.

Keywords: Big data, diabetes mellitus, graph, GNNs, node, protein interactions.

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List of Participants

Contributed Speaker(*)
Invited Speaker(**)

Efim Zelmanov**	San Diego, USA
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Jun Zhang**	Ann Arbor, USA
Ngaiming Mok**	Hong Kong, China
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Adara M. Blaga	Timisoara , Romania
Adel Delloum *	Mascara , Algeria
Ahmed Mohammed Cherif *	Mascara , Algeria
Ahmet Duman	Konya , Turkey
Alfi Y. Zakiyyah *	Jakarta , Indonesia
Ali Aral	Kirikkale , Turkey
Ali Aydogdu *	Izmir , Turkey
Ali Bashan *	Zonguldak , Turkey
Ali Demirci *	Istanbul , Turkey
Ali Hikmet Deger	Trabzon , Turkey
Ali Moradzadeh-Dehkordi	Isfahan , Iran
Alper Vural *	Istanbul , Turkey
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Anil Zenginoglu *	Maryland , USA
Anil Sharma	Punjab , India
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Ayhan Erciyes	Aksaray , Turkey
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Ayse Nur Altuntas Tankul *	Karabuk , Turkey
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Basak Aldemir	Afyon , Turkey
Bayan Kutum	Karaganda , Kazakhstan
Bayour Benaoumeur *	Mascara , Algeria
Bedia Merih Ozcetin *	Istanbul , Turkey

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