Journal of Inequalities and Applications
a SpringerOpen Journal

RESEARCH Open Access

The multiplicative Zagreb indices of graph operations

Kinkar C Das[1], Aysun Yurttas[2], Muge Togan[2], Ahmet Sinan Cevik[3] and Ismail Naci Cangul[2*]

*Correspondence:
cangul@uludag.edu.tr
[2]Department of Mathematics, Faculty of Arts and Science, Uludağ University, Gorukle Campus, Bursa, 16059, Turkey
Full list of author information is available at the end of the article

Abstract

Recently, Todeschini *et al.* (Novel Molecular Structure Descriptors - Theory and Applications I, pp. 73-100, 2010), Todeschini and Consonni (MATCH Commun. Math. Comput. Chem. 64:359-372, 2010) have proposed the multiplicative variants of ordinary Zagreb indices, which are defined as follows:

$$\prod_1 = \prod_1(G) = \prod_{v\in V(G)} d_G(v)^2, \qquad \prod_2 = \prod_2(G) = \prod_{uv\in E(G)} d_G(u)d_G(v).$$

These two graph invariants are called *multiplicative Zagreb indices* by Gutman (Bull. Soc. Math. Banja Luka 18:17-23, 2011). In this paper the upper bounds on the multiplicative Zagreb indices of the join, Cartesian product, corona product, composition and disjunction of graphs are derived and the indices are evaluated for some well-known graphs.

MSC: 05C05; 05C90; 05C07

Keywords: graph; multiplicative Zagreb index; graph operations

1 Introduction

Throughout this paper, we consider simple graphs which are finite, indirected graphs without loops and multiple edges. Suppose G is a graph with a vertex set $V(G)$ and an edge set $E(G)$. For a graph G, the degree of a vertex v is the number of edges incident to v and is denoted by $d_G(v)$. A topological index $\mathrm{Top}(G)$ of a graph G is a number with the property that for every graph H isomorphic to G, $\mathrm{Top}(H) = \mathrm{Top}(G)$. Recently, Todeschini *et al.* [1, 2] have proposed the multiplicative variants of ordinary Zagreb indices, which are defined as follows:

$$\prod_1 = \prod_1(G) = \prod_{v\in V(G)} d_G(v)^2, \qquad \prod_2 = \prod_2(G) = \prod_{uv\in E(G)} d_G(u)d_G(v).$$

Mathematical properties and applications of multiplicative Zagreb indices are reported in [1–6]. Mathematical properties and applications of multiplicative sum Zagreb indices are reported in [7]. For other undefined notations and terminology from graph theory, the readers are referred to [8].

In [9, 10], Khalifeh *et al.* computed some exact formulae for the hyper-Wiener index and Zagreb indices of the join, Cartesian product, composition, disjunction and symmetric

Springer

© 2013 Das et al.; licensee Springer. This is an Open Access article distributed under the terms of the Creative Commons Attribution License (http://creativecommons.org/licenses/by/2.0), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

difference of graphs. Some more properties and applications of graph products can be seen in the classical book [11].

In this paper, we give some upper bounds for the multiplicative Zagreb index of various graph operations such as join, corona product, Cartesian product, composition, disjunction, *etc.* Moreover, computations are done for some well-known graphs.

2 Multiplicative Zagreb index of graph operations

We begin this section with two standard inequalities as follows.

Lemma 1 (AM-GM inequality) *Let $x_1, x_2, \ldots, x_n$ be nonnegative numbers. Then*

$$\frac{x_1 + x_2 + \cdots + x_n}{n} \geq \sqrt[n]{x_1 x_2 \cdots x_n} \tag{1}$$

holds with equality if and only if all the x_k's are equal.

Lemma 2 (Weighted AM-GM inequality) *Let $x_1, x_2, \ldots, x_n$ be nonnegative numbers and also let $w_1, w_2, \ldots, w_n$ be nonnegative weights. Set $w = w_1 + w_2 + \cdots + w_n$. If $w > 0$, then the inequality*

$$\frac{w_1 x_1 + w_2 x_2 + \cdots + w_n x_n}{w} \geq \sqrt[w]{x_1^{w_1} x_2^{w_2} \cdots x_n^{w_n}} \tag{2}$$

holds with equality if and only if all the x_k with $w_k > 0$ are equal.

Let G_1 and G_2 be two graphs with n_1 and n_2 vertices and m_1 and m_2 edges, respectively. The join $G_1 \vee G_2$ of graphs G_1 and G_2 with disjoint vertex sets $V(G_1)$ and $V(G_2)$ and edge sets $E(G_1)$ and $E(G_2)$ is the graph union $G_1 \cup G_2$ together with all the edges joining $V(G_1)$ and $V(G_2)$. Thus, for example, $\overline{K}_p \vee \overline{K}_q = K_{p,q}$, the complete bipartite graph. We have $|V(G_1 \vee G_2)| = n_1 + n_2$ and $|E(G_1 \vee G_2)| = m_1 + m_2 + n_1 n_2$.

Theorem 1 *Let G_1 and G_2 be two graphs. Then*

$$\prod_1 (G_1 \vee G_2) \leq \left[\frac{M_1(G_1) + 4m_1 n_2 + n_1 n_2^2}{n_1}\right]^{n_1} \times \left[\frac{M_1(G_2) + 4n_1 m_2 + n_2 n_1^2}{n_2}\right]^{n_2} \tag{3}$$

and

$$\prod_2 (G_1 \vee G_2) \leq \left[\frac{M_2(G_1) + n_2 M_1(G_1) + m_1 n_2^2}{m_1}\right]^{m_1} \times \left[\frac{M_2(G_2) + n_1 M_1(G_2) + m_2 n_1^2}{m_2}\right]^{m_2} \times \left[\frac{4m_1 m_2 + 2n_1 n_2(m_1 + m_2) + (n_1 n_2)^2}{n_1 n_2}\right]^{n_1 n_2}, \tag{4}$$

where n_1 and n_2 are the numbers of vertices of G_1 and G_2, and m_1, m_2 are the numbers of edges of G_1 and G_2, respectively. Moreover, the equality holds in (3) *if and only if both G_1 and G_2 are regular graphs, that is, $G_1 \vee G_2$ is a regular graph and the equality holds in* (4) *if and only if both G_1 and G_2 are regular graphs, that is, $G_1 \vee G_2$ is a regular graph.*

Proof Now,

$$\begin{aligned}\prod_1(G_1 \vee G_2) &= \prod_{(u_i,v_j)\in V(G_1\vee G_2)} d_{G_1\vee G_2}(u_i,v_j)^2 \\ &= \prod_{u_i\in V(G_1)} \big(d_{G_1}(u_i)+n_2\big)^2 \prod_{v_j\in V(G_2)} \big(d_{G_2}(v_j)+n_1\big)^2 \\ &= \prod_{u_i\in V(G_1)} \big(d_{G_1}(u_i)^2+2n_2d_{G_1}(u_i)+n_2^2\big) \prod_{v_j\in V(G_2)} \big(d_{G_2}(v_j)^2+2n_1d_{G_2}(v_j)+n_1^2\big)\end{aligned}$$

and by (1) this above equality is actually less than or equal to

$$\begin{aligned}&\le \left[\frac{\sum_{u_i\in V(G_1)}(d_{G_1}(u_i)^2+2n_2d_{G_1}(u_i)+n_2^2)}{n_1}\right]^{n_1} \\ &\quad\times\left[\sum_{v_j\in V(G_2)}\frac{(d_{G_2}(v_j)^2+2n_1d_{G_2}(v_j)+n_1^2)}{n_2}\right]^{n_2} \\ &= \left[\frac{M_1(G_1)+4m_1n_2+n_1n_2^2}{n_1}\right]^{n_1}\times\left[\frac{M_1(G_2)+4n_1m_2+n_2n_1^2}{n_2}\right]^{n_2}.\end{aligned}$$

Moreover, the above equality holds if and only if

$$d_{G_1}(u_i)^2+2n_2d_{G_1}(u_i)+n_2^2 = d_{G_1}(u_k)^2+2n_2d_{G_1}(u_k)+n_2^2 \quad \big(u_i,u_k\in V(G_1)\big)$$

and

$$d_{G_2}(v_j)^2+2n_1d_{G_2}(v_j)+n_1^2 = d_{G_2}(v_\ell)^2+2n_1d_{G_2}(v_\ell)+n_1^2 \quad \big(v_j,v_\ell\in V(G_2)\big)$$

(by Lemma 1), that is, for $u_i,u_k\in V(G_1)$ and $v_j,v_\ell\in V(G_2)$,

$$\big(d_{G_1}(u_i)-d_{G_1}(u_k)\big)\big(d_{G_1}(u_i)+d_{G_1}(u_k)+2n_2\big)$$

and

$$\big(d_{G_2}(v_j)-d_{G_2}(v_\ell)\big)\big(d_{G_2}(v_j)+d_{G_2}(v_\ell)+2n_1\big).$$

That is, for $u_i,u_k\in V(G_1)$ and $v_j,v_\ell\in V(G_2)$, we get $d_{G_1}(u_i)=d_{G_1}(u_k)$ and $d_{G_2}(v_j)=d_{G_2}(v_\ell)$. Hence the equality holds in (3) if and only if both G_1 and G_2 are regular graphs, that is, $G_1\vee G_2$ is a regular graph.

Now, since

$$\prod_2(G_1\vee G_2) = \prod_{(u_i,v_j)(u_k,v_\ell)\in E(G_1\vee G_2)} d_{G_1\vee G_2}(u_i,v_j)d_{G_1\vee G_2}(u_k,v_\ell),$$

we then obtain

$$\begin{aligned}&= \prod_{u_iu_k\in E(G_1)}\big(d_{G_1}(u_i)+n_2\big)\big(d_{G_1}(u_k)+n_2\big)\prod_{v_jv_\ell\in E(G_2)}\big(d_{G_2}(v_j)+n_1\big)\big(d_{G_2}(v_\ell)+n_1\big) \\ &\quad\times\prod_{u_i\in V(G_1),v_j\in V(G_2)}\big(d_{G_1}(u_i)+n_2\big)\big(d_{G_2}(v_j)+n_1\big)\end{aligned}$$

and by (1)

$$\begin{aligned}
&\leq \left[\frac{\sum_{u_iu_k\in E(G_1)}(d_{G_1}(u_i)d_{G_1}(u_k)+n_2(d_{G_1}(u_i)+d_{G_1}(u_k))+n_2^2)}{m_1}\right]^{m_1}\\
&\quad\times\left[\frac{\sum_{v_jv_\ell\in E(G_2)}(d_{G_2}(v_j)d_{G_2}(v_\ell)+n_1(d_{G_2}(v_j)+d_{G_2}(v_\ell))+n_1^2)}{m_2}\right]^{m_2}\\
&\quad\times\left[\frac{\sum_{u_i\in V(G_1),v_j\in V(G_2)}(d_{G_1}(u_i)d_{G_2}(v_j)+n_2d_{G_2}(v_j)+n_1d_{G_1}(u_i)+n_1n_2)}{n_1n_2}\right]^{n_1n_2}. \qquad (5)
\end{aligned}$$

However, from the last inequality, we get

$$\begin{aligned}
&=\left[\frac{M_2(G_1)+n_2M_1(G_1)+m_1n_2^2}{m_1}\right]^{m_1}\times\left[\frac{M_2(G_2)+n_1M_1(G_2)+m_2n_1^2}{m_2}\right]^{m_2}\\
&\quad\times\left[\frac{\sum_{u_i\in V(G_1)}d_i\sum_{v_j\in V(G_2)}d_j^*+n_1n_2\sum_{v_i\in V(G_1)}d_i+n_1n_2\sum_{v_j\in V(G_2)}d_j^*+n_1^2n_2^2}{n_1n_2}\right]^{n_1n_2}\\
&=\left[\frac{M_2(G_1)+n_2M_1(G_1)+m_1n_2^2}{m_1}\right]^{m_1}\times\left[\frac{M_2(G_2)+n_1M_1(G_2)+m_2n_1^2}{m_2}\right]^{m_2}\\
&\quad\times\left[\frac{4m_1m_2+2n_1n_2(m_1+m_2)+(n_1n_2)^2}{n_1n_2}\right]^{n_1n_2}.
\end{aligned}$$

Furthermore, for both connected graphs G_1 and G_2, the equality holds in (5) iff

$$d_{G_1}(u_i)d_{G_1}(u_r)+n_2\big(d_{G_1}(u_i)+d_{G_1}(u_r)\big)+n_2^2=d_{G_1}(u_i)d_{G_1}(u_k)+n_2\big(d_{G_1}(u_i)+d_{G_1}(u_k)\big)+n_2^2$$

for any $u_iu_r, u_iu_k\in E(G_1)$; and

$$d_{G_2}(v_j)d_{G_2}(v_r)+n_1\big(d_{G_2}(v_j)+d_{G_2}(v_r)\big)+n_1^2=d_{G_2}(v_j)d_{G_2}(v_\ell)+n_1\big(d_{G_2}(v_j)+d_{G_2}(v_\ell)\big)+n_1^2$$

for any $v_jv_r, v_jv_\ell\in E(G_2)$ as well as

$$\begin{aligned}
&d_{G_1}(u_i)d_{G_2}(v_j)+n_2d_{G_2}(v_j)+n_1d_{G_1}(u_i)+n_1n_2\\
&\quad=d_{G_1}(u_i)d_{G_2}(v_\ell)+n_2d_{G_2}(v_\ell)+n_1d_{G_1}(u_i)+n_1n_2
\end{aligned}$$

for any $u_i\in V(G_1)$, $v_j, v_\ell\in V(G_2)$; and

$$\begin{aligned}
&d_{G_1}(u_i)d_{G_2}(v_j)+n_2d_{G_2}(v_j)+n_1d_{G_1}(u_i)+n_1n_2\\
&\quad=d_{G_1}(u_k)d_{G_2}(v_j)+n_2d_{G_2}(v_j)+n_1d_{G_1}(u_k)+n_1n_2
\end{aligned}$$

for any $v_j\in V(G_2)$, $u_i, u_k\in V(G_1)$ by Lemma 1. Thus one can easily see that the equality holds in (5) if and only if for $u_i, u_k\in V(G_1)$ and $v_j, v_\ell\in V(G_2)$,

$$d_{G_1}(u_i)=d_{G_1}(u_k)\quad\text{and}\quad d_{G_2}(v_j)=d_{G_2}(v_\ell).$$

Hence the equality holds in (4) if and only if both G_1 and G_2 are regular graphs, that is, $G_1\vee G_2$ is a regular graph. □

Example 1 Consider two cycle graphs C_p and C_q. We thus have

$$\prod_1(C_p \vee C_q) = (p+2)^{2q}(q+2)^{2p} \quad \text{and} \quad \prod_2(C_p \vee C_q) = (p+2)^{(p+2)q}(q+2)^{(q+2)p}.$$

The *Cartesian product* $G_1 \boxtimes G_2$ of graphs G_1 and G_2 has the vertex set $V(G_1 \times G_2) = V(G_1) \times V(G_2)$ and $(u_i, v_j)(u_k, v_\ell)$ is an edge of $G_1 \boxtimes G_2$ if

either $u_i = u_k$ and $v_j v_\ell \in E(G_2)$,

or $u_i u_k \in E(G_1)$ and $v_j = v_\ell$.

Theorem 2 *Let G_1 and G_2 be two connected graphs. Then*

(i)

$$\prod_1(G_1 \boxtimes G_2) \le \left[\frac{n_2 M_1(G_1) + n_1 M_1(G_2) + 8m_1m_2}{n_1n_2}\right]^{n_1n_2}. \tag{6}$$

The equality holds in (6) *if and only if $G_1 \boxtimes G_2$ is a regular graph.*

(ii)

$$\prod_2(G_1 \boxtimes G_2) \le \frac{1}{(2n_1m_2)^{2n_1m_2}}\left(n_1M_1(G_2) + 4m_1m_2\right)^{2n_1m_2} \times \frac{1}{(2n_2m_1)^{2n_2m_1}}\left(n_2M_1(G_1) + 4m_1m_2\right)^{2n_2m_1}. \tag{7}$$

Moreover, the equality holds in (7) *if and only if $G_1 \boxtimes G_2$ is a regular graph.*

Proof By the definition of the first multiplicative Zagreb index, we have

$$\prod_1(G_1 \boxtimes G_2) = \prod_{(u_i,v_j)\in V(G_1\boxtimes G_2)}\left(d_{G_1}(u_i) + d_{G_2}(v_j)\right)^2 = \prod_{u_i\in V(G_1)}\prod_{v_j\in V(G_2)}\left(d_{G_1}(u_i) + d_{G_2}(v_j)\right)^2.$$

On the other hand, by (1)

$$\le \left[\frac{\sum_{u_i\in V(G_1)}\sum_{v_j\in V(G_2)}(d_{G_1}(u_i)^2 + d_{G_2}(v_j)^2 + 2d_{G_1}(u_i)d_{G_2}(v_j))}{n_1n_2}\right]^{n_1n_2}. \tag{8}$$

But as $\sum_{u_i\in V(G_1)} d_{G_1}(u_i)^2 = M_1(G_1)$ and $\sum_{v_j\in V(G_2)} d_{G_2}(v_j)^2 = M_1(G_2)$, the last statement in (8) is less than or equal to

$$\le \left[\frac{\sum_{u_i\in V(G_1)}(d_{G_1}(u_i)^2\sum_{v_j\in V(G_2)}1 + \sum_{v_j\in V(G_2)}d_{G_2}(v_j)^2 + 2d_{G_1}(u_i)\sum_{v_j\in V(G_2)}d_{G_2}(v_j))}{n_1n_2}\right]^{n_1n_2}$$

which equals to

$$\left[\frac{n_2M_1(G_1) + n_1M_1(G_2) + 8m_1m_2}{n_1n_2}\right]^{n_1n_2}.$$

Moreover, the equality holds in (8) if and only if $d_{G_1}(u_i) + d_{G_2}(v_j) = d_{G_1}(u_k) + d_{G_2}(v_\ell)$ for any $(u_i, v_j), (u_k, v_\ell) \in V(G_1 \boxtimes G_2)$ by Lemma 1. Since both G_1 and G_2 are connected graphs, one can easily see that the equality holds in (8) if and only if $d_{G_1}(u_i) = d_{G_1}(u_k)$, $u_i, u_k \in V(G_1)$ and $d_{G_2}(v_j) = d_{G_2}(v_\ell)$, $v_j, v_\ell \in V(G_2)$. Hence the equality holds in (6) if and only if both G_1 and G_2 are regular graphs, that is, $G_1 \boxtimes G_2$ is a regular graph. This completes the first part of the proof.

By the definition of the second multiplicative Zagreb index, we have

$$\prod_2(G_1 \boxtimes G_2) = \prod_{(u_i,v_j)(u_k,v_\ell)\in E(G_1\boxtimes G_2)} \big(d_{G_1}(u_i) + d_{G_2}(v_j)\big)\big(d_{G_1}(u_k) + d_{G_2}(v_\ell)\big).$$

This actually can be written as

$$\begin{aligned}\prod_2(G_1 \boxtimes G_2) = {} & \prod_{u_i\in V(G_1)} \prod_{v_jv_\ell\in E(G_2)} \big(d_{G_1}(u_i) + d_{G_2}(v_j)\big)\big(d_{G_1}(u_i) + d_{G_2}(v_\ell)\big) \\ & \times \prod_{v_j\in V(G_2)} \prod_{u_iu_k\in E(G_1)} \big(d_{G_1}(u_i) + d_{G_2}(v_j)\big)\big(d_{G_1}(u_k) + d_{G_2}(v_j)\big)\end{aligned}$$

or, equivalently,

$$\begin{aligned}\prod_2(G_1 \boxtimes G_2) = {} & \prod_{u_i\in V(G_1)} \prod_{v_j\in V(G_2)} \big(d_{G_1}(u_i) + d_{G_2}(v_j)\big)^{d_{G_2}(v_j)} \\ & \times \prod_{v_j\in V(G_2)} \prod_{u_i\in V(G_1)} \big(d_{G_1}(u_i) + d_{G_2}(v_j)\big)^{d_{G_1}(u_i)}.\end{aligned}$$

After that, by (2) we get

$$\begin{aligned}\prod_2(G_1 \boxtimes G_2) \le {} & \prod_{u_i\in V(G_1)} \left[\frac{\sum_{v_j\in V(G_2)} d_{G_2}(v_j)(d_{G_1}(u_i) + d_{G_2}(v_j))}{2m_2}\right]^{2m_2} \\ & \times \prod_{v_j\in V(G_2)} \left[\frac{\sum_{u_i\in V(G_1)} d_{G_1}(u_i)(d_{G_1}(u_i) + d_{G_2}(v_j))}{2m_1}\right]^{2m_1}. \qquad (9)\end{aligned}$$

Moreover, since

$$\sum_{u_i\in V(G_1)} d_{G_1}(u_i) = 2m_1, \qquad \sum_{v_j\in V(G_2)} d_{G_2}(v_j) = 2m_2 \quad \text{and}$$

$$\sum_{u_i\in V(G_1)} d_{G_1}(u_i)^2 = M_1(G_1), \qquad \sum_{v_j\in V(G_2)} d_{G_2}(v_j)^2 = M_1(G_2).$$

By (1) the final statement in (9) becomes

$$\begin{aligned}& = \prod_{u_i\in V(G_1)} \left[\frac{M_1(G_2) + 2m_2 d_{G_1}(u_i)}{2m_2}\right]^{2m_2} \times \prod_{v_j\in V(G_2)} \left[\frac{M_1(G_1) + 2m_1 d_{G_2}(v_j)}{2m_1}\right]^{2m_1} \\ & \le \frac{1}{(2m_2)^{2n_1m_2}} \left[\frac{\sum_{u_i\in V(G_1)}(M_1(G_2) + 2m_2 d_{G_1}(u_i))}{n_1}\right]^{2n_1m_2}\end{aligned}$$

$$\times \frac{1}{(2m_1)^{2n_2m_1}}\left[\frac{\sum_{v_j\in V(G_2)}(M_1(G_1)+2m_1d_{G_2}(v_j))}{n_2}\right]^{2n_2m_1} \tag{10}$$
$$= \frac{1}{(2n_1m_2)^{2n_1m_2}}\big(n_1M_1(G_2)+4m_1m_2\big)^{2n_1m_2}$$
$$\times \frac{1}{(2n_2m_1)^{2n_2m_1}}\big(n_2M_1(G_1)+4m_1m_2\big)^{2n_2m_1}.$$

Hence the second part of the proof is over.

The equality holds in (9) and (10) if and only if $d_{G_2}(v_j) = d_{G_2}(v_\ell)$ for any $v_j, v_\ell \in V(G_2)$ and $d_{G_1}(u_i) = d_{G_1}(u_k)$ for any $u_i, u_k \in V(G_1)$ by Lemmas 1 and 2. Hence the equality holds in (7) if and only if both G_1 and G_2 are regular graphs, that is, $G_1 \boxtimes G_2$ is a regular graph. This completes the proof. □

Example 2 Consider a cycle graph C_p and a complete graph K_q. We thus have

$$\prod_1(C_p \boxtimes K_q) = (q+1)^{2pq} \quad \text{and} \quad \prod_2(C_p \boxtimes K_q) = (q+1)^{(q+1)pq}.$$

The *corona product* $G_1 \circ G_2$ of two graphs G_1 and G_2 is defined to be the graph Γ obtained by taking one copy of G_1 (which has n_1 vertices) and n_1 copies of G_2, and then joining the ith vertex of G_1 to every vertex in the ith copy of G_2, $i = 1, 2, \ldots, n_1$.

Let $G_1 = (V, E)$ and $G_2 = (V, E)$ be two graphs such that $V(G_1) = \{u_1, u_2, \ldots, u_{n_1}\}$, $|E(G_1)| = m_1$ and $V(G_2) = \{v_1, v_2, \ldots, v_{n_2}\}$, $|E(G_2)| = m_2$. Then it follows from the definition of the corona product that $G_1 \circ G_2$ has $n_1(1 + n_2)$ vertices and $m_1 + n_1m_2 + n_1n_2$ edges, where $V(G_1 \circ G_2) = \{(u_i, v_j), i = 1, 2, \ldots, n_1; j = 0, 1, 2, \ldots, n_2\}$ and $E(G_1 \circ G_2) = \{((u_i, v_0), (u_k, v_0)), (u_i, u_k) \in E(G_1)\} \cup \{((u_i, v_j), (u_i, v_\ell)), (v_j, v_\ell) \in E(G_2), i = 1, 2, \ldots, n_1\} \cup \{((u_i, v_0), (u_i, v_\ell)), \ell = 1, 2, \ldots, n_2, i = 1, 2, \ldots, n_1\}$. It is clear that if G_1 is connected, then $G_1 \circ G_2$ is connected, and in general $G_1 \circ G_2$ is not isomorphic to $G_2 \circ G_1$.

Theorem 3 *The first and second multiplicative Zagreb indices of the corona product are computed as follows*:

(i)

$$\prod_1(G_1 \circ G_2) \le \frac{1}{n_1^{n_1}n_2^{n_1n_2}}M_1(G_1)^{n_1}\big(M_1(G_2)+4m_2+n_2\big)^{n_1n_2}, \tag{11}$$

(ii)

$$\prod_2(G_1 \circ G_2) \le \left[\frac{M_2(G_1)+n_2M_1(G_1)+n_2^2}{m_1}\right]^{m_1}\left[\frac{M_2(G_2)+M_1(G_2)+1}{m_2}\right]^{n_1m_2}$$
$$\times \left[\frac{4m_1m_2+n_1n_2^2+2m_1n_2+2m_2n_1n_2}{n_1n_2}\right]^{n_1n_2}, \tag{12}$$

where $M_1(G_i)$ and $M_2(G_i)$ are the first and second Zagreb indices of G_i, where $i = 1, 2$, respectively. Moreover, both equalities in (11) *and* (12) *hold if and only if $G_1 \circ G_2$ is a regular graph.*

Proof By the definition of the first multiplicative Zagreb index, we have

$$\begin{aligned}
\prod_1(G_1 \circ G_2) &= \prod_{(u_i,v_j)\in V(G_1\circ G_2)} d_{G_1\circ G_2}(u_i,v_j)^2 \\
&= \prod_{u_i\in V(G_1)} \big(d_{G_1}(u_i)+n_2\big)^2 \prod_{u_i\in V(G_1)}\prod_{v_j\in V(G_2)} \big(d_{G_2}(v_j)+1\big)^2 \\
&= \prod_{u_i\in V(G_1)} \big(d_{G_1}(u_i)^2 + 2n_2 d_{G_1}(u_i) + n_2^2\big) \\
&\quad \times \Bigg[\prod_{v_j\in V(G_2)} \big(d_{G_2}(v_j)^2 + 2d_{G_2}(v_j)+1\big)^2\Bigg]^{n_1} \\
&\le \Bigg[\frac{\sum_{u_i\in V(G_1)}(d_{G_1}(u_i)^2 + 2n_2 d_{G_1}(u_i) + n_2^2)}{n_1}\Bigg]^{n_1} \\
&\quad \times \Bigg[\frac{\sum_{v_j\in V(G_2)}(d_{G_2}(v_j)^2 + 2d_{G_2}(v_j)+1)}{n_2}\Bigg]^{n_1n_2} \quad \text{by (1)} \qquad (13) \\
&= \frac{1}{n_1^{n_1}n_2^{n_1n_2}}\big(M_1(G_1)+4n_2m_1+n_1n_2^2\big)^{n_1}\big(M_1(G_2)+4m_2+n_2\big)^{n_1n_2}.
\end{aligned}$$

The equality holds in (13) if and only if $d_{G_1}(u_i) = d_{G_1}(u_k)$, $u_i, u_k \in V(G_1)$ and $d_{G_2}(v_j) = d_{G_2}(v_\ell)$, $v_j, v_\ell \in V(G_2)$, that is, both G_1 and G_2 are regular graphs, that is, $G_1 \circ G_2$ is a regular graph.

By the definition of the second multiplicative Zagreb index, we have

$$\begin{aligned}
\prod_2(G_1 \circ G_2) &= \prod_{(u_i,v_j)(u_k,v_\ell)\in E(G_1\circ G_2)} d_{G_1\circ G_2}(u_i,v_j)d_{G_1\circ G_2}(u_k,v_\ell) \\
&= \prod_{u_iu_k\in E(G_1)} \big(d_{G_1}(u_i)+n_2\big)\big(d_{G_1}(u_k)+n_2\big) \\
&\quad \times \prod_{u_i\in V(G_1)}\prod_{v_j\in V(G_2)} \big(d_{G_1}(u_i)+n_2\big)\big(d_{G_2}(v_j)+1\big) \\
&\quad \times \prod_{u_i\in V(G_1)}\prod_{v_jv_\ell\in E(G_2)} \big(d_{G_2}(v_j)+1\big)\big(d_{G_2}(v_\ell)+1\big) \\
&= \prod_{u_iu_k\in E(G_1)} \big(d_{G_1}(u_i)d_{G_1}(u_k) + n_2\big(d_{G_1}(u_i)+d_{G_1}(u_k)\big) + n_2^2\big) \\
&\quad \times \Bigg[\prod_{u_i\in V(G_1)} \big(d_{G_1}(u_i)+n_2\big)\Bigg]^{n_2}\Bigg[\prod_{v_j\in V(G_2)} \big(d_{G_1}(v_j)+1\big)\Bigg]^{n_1} \\
&\quad \times \Bigg[\prod_{v_jv_\ell\in E(G_2)} \big(d_{G_2}(v_j)d_{G_2}(v_\ell) + \big(d_{G_2}(v_j)+d_{G_2}(v_\ell)\big)+1\big)\Bigg]^{n_1} \\
&\le \Bigg[\frac{M_2(G_1)+n_2M_1(G_1)+n_2^2m_1}{m_1}\Bigg]^{m_1} \times \Bigg[\frac{2m_1+n_1n_2}{n_1}\Bigg]^{n_1n_2} \\
&\quad \times \Bigg[\frac{2m_2+n_2}{n_2}\Bigg]^{n_1n_2} \times \Bigg[\frac{M_2(G_2)+M_1(G_2)+m_2}{m_2}\Bigg]^{n_1m_2} \quad \text{by (1).}
\end{aligned}$$

The above equality holds if and only if $d_{G_1}(u_i) = d_{G_1}(u_k)$ for any $u_i, u_k \in V(G_1)$ and $d_{G_2}(v_j) = d_{G_2}(v_\ell)$ for any $v_j, v_\ell \in V(G_2)$, that is, both G_1 and G_2 are regular graphs, which implies that $G_1 \circ G_2$ is a regular graph. This completes the proof. □

Example 3 $\prod_1(C_p \circ K_q) = q^{2pq}(q+2)^{2p}$ and $\prod_2(C_p \circ K_q) = q^{pq^2}(q+2)^{p(q+2)}$.

The *composition* (also called *lexicographic product* [12]) $G = G_1[G_2]$ of graphs G_1 and G_2 with disjoint vertex sets $V(G_1)$ and $V(G_2)$ and edge sets $E(G_1)$ and $E(G_2)$ is the graph with a vertex set $V(G_1) \times V(G_2)$ and (u_i, v_j) is adjacent to (u_k, v_ℓ) whenever

either u_i is adjacent to u_k,

or $u_i = u_k$ and v_j is adjacent to v_ℓ.

Theorem 4 *The first and second multiplicative Zagreb indices of the composition $G_1[G_2]$ of graphs G_1 and G_2 are bounded above as follows*:

(i)

$$\prod_1\big(G_1[G_2]\big) \le \frac{1}{(n_1n_2)^{n_1n_2}}\big[n_2^3M_1(G_1) + 8n_2m_1m_2 + n_1M_1(G_2)\big]^{n_1n_2}, \tag{14}$$

(ii)

$$\begin{aligned}\prod_2\big(G_1[G_2]\big) \le{}& \frac{1}{(n_1m_2)^{n_1m_2}}\big[m_2n_2^2M_1(G_1) + 2n_2m_1M_1(G_2) + n_1M_2(G_2)\big]^{n_1m_2}\\ &\times \frac{1}{(n_2m_1)^{m_1n_2^2}}\big[n_2^3M_2(G_1) + m_1M_1(G_2) + 2m_2n_2M_1(G_1)\big]^{n_2^2m_1},\end{aligned} \tag{15}$$

where $M_1(G_i)$ and $M_2(G_i)$ are the first and second Zagreb indices of G_i, where $i = 1, 2$. Moreover, the equalities in (14) *and* (15) *hold if and only if $G_1 \circ G_2$ is a regular graph*.

Proof By the definition of the first multiplicative Zagreb index, we have

$$\begin{aligned}&\prod_1\big(G_1[G_2]\big)\\ &= \prod_{(u_i,v_j)\in V(G_1[G_2])} d_{G_1[G_2]}(u_i, v_j)^2\\ &= \prod_{u_i\in V(G_1)}\prod_{v_j\in V(G_2)}\big(d_{G_1}(u_i)n_2 + d_{G_2}(v_j)\big)^2\\ &\le \left[\frac{\sum_{u_i\in V(G_1)}\sum_{v_j\in V(G_2)}(n_2^2d_{G_1}(u_i)^2 + 2n_2d_{G_1}(u_i)d_{G_2}(v_j) + d_{G_2}(v_j)^2)}{n_1n_2}\right]^{n_1n_2}\\ &= \frac{1}{(n_1n_2)^{n_1n_2}}\big[n_2^3M_1(G_1) + 8n_2m_1m_2 + n_1M_1(G_2)\big]^{n_1n_2}.\end{aligned} \tag{16}$$

The equality holds in (16) if and only if $d_{G_1}(u_i) = d_{G_1}(u_k)$, $u_i, u_k \in V(G_1)$ and $d_{G_2}(v_j) = d_{G_2}(v_\ell)$, $v_j, v_\ell \in V(G_2)$ (by Lemma 1), that is, both G_1 and G_2 are regular graphs, that is, $G_1 \circ G_2$ is a regular graph.

By the definition of the second multiplicative Zagreb index, we have

$$\begin{aligned}
&\prod_2(G_1[G_2]) \\
&= \prod_{(u_i,v_j)(u_k,v_\ell)\in E(G_1[G_2])} d_{G_1\circ G_2}(u_i,v_j)d_{G_1[G_2]}(u_k,v_\ell) \\
&= \prod_{u_i\in V(G_1)}\prod_{v_jv_\ell\in E(G_2)} \big(d_{G_1}(u_i)n_2 + d_{G_2}(v_j)\big)\big(d_{G_1}(u_i)n_2 + d_{G_2}(v_\ell)\big) \\
&\quad\times \prod_{u_iu_k\in E(G_1)}\prod_{v_j\in V(G_2)} \big[\big(d_{G_1}(u_i)n_2 + d_{G_2}(v_j)\big)\big(d_{G_1}(u_k)n_2 + d_{G_2}(v_j)\big)\big]^{n_2} \\
&\le \prod_{u_i\in V(G_1)}\left[\frac{m_2n_2^2d_{G_1}(u_i)^2 + n_2d_{G_1}(u_i)M_1(G_2) + M_2(G_2)}{m_2}\right]^{m_2} \\
&\quad\times \prod_{u_iu_k\in E(G_1)}\left[\frac{n_2^3d_{G_1}(u_i)d_{G_1}(u_k) + M_1(G_2) + 2m_2n_2(d_{G_1}(u_i) + d_{G_1}(u_k))}{n_2}\right]^{n_2^2} \qquad (17) \\
&\le \frac{1}{m_2^{n_1m_2}}\left[\frac{m_2n_2^2M_1(G_1) + 2n_2m_1M_1(G_2) + n_1M_2(G_2)}{n_1}\right]^{n_1m_2} \\
&\quad\times \frac{1}{(n_2)^{m_1n_2^2}}\left[\frac{n_2^3M_2(G_1) + m_1M_1(G_2) + 2m_2n_2M_1(G_1)}{m_1}\right]^{n_2^2m_1}, \qquad (18)
\end{aligned}$$

which gives the required result in (15).

The equality holds in (17) and (18) if and only if $d_{G_1}(u_i) = d_{G_1}(u_k)$, $u_i, u_k \in V(G_1)$ and $d_{G_2}(v_j) = d_{G_2}(v_\ell)$, $v_j, v_\ell \in V(G_2)$ (by Lemma 1), that is, both G_1 and G_2 are regular graphs, that is, $G_1 \circ G_2$ is a regular graph. □

Example 4 $\prod_1(C_p[C_q]) = 2^{2pq}(q+1)^{2pq}$ and $\prod_2(C_p[C_q]) = 2^{2pq(q+1)}(q+1)^{2pq(q+1)}$.

The *disjunction* $G_1 \otimes G_2$ of graphs G_1 and G_2 is the graph with a vertex set $V(G_1) \times V(G_2)$ and (u_i, v_j) is adjacent to (u_k, v_ℓ) whenever $u_iu_k \in E(G_1)$ or $v_jv_\ell \in E(G_2)$.

Theorem 5 *The first and second multiplicative Zagreb indices of the disjunction are computed as follows*:

(i)

$$\begin{aligned}
\prod_1(G_1 \otimes G_2) \le \frac{1}{(n_1n_2)^{n_1n_2}} &\big[n_2^3M_1(G_1) + n_1^3M_1(G_2) + M_1(G_1)M_1(G_2) \\
&+ 8n_1n_2m_1m_2 - 4n_1m_1M_1(G_2) - 4n_2m_2M_1(G_1)\big]^{n_1n_2}, \qquad (19)
\end{aligned}$$

(ii)

$$\begin{aligned}
&\prod_2(G_1 \otimes G_2) \\
&\le \left[\frac{M_1(G_1)(n_2^3 + M_1(G_2) - 4n_2m_2) + M_1(G_2)(n_1^2 - 4n_1m_1) + 8n_1n_2m_1m_2}{Q}\right]^Q, \qquad (20)
\end{aligned}$$

where $Q = \sum_{u_i \in V(G_1)} \sum_{v_j \in V(G_2)} P = 2(n_2^2 m_1 + n_1^2 m_2 - 2m_1 m_2)$ *and* $M_1(G_i)$ *is the first Zagreb index of* G_i, $i = 1, 2$. *Moreover, the equalities in* (19) *and* (20) *hold if and only if* $G_1 \circ G_2$ *is a regular graph*.

Proof We have $d_{G_1 \otimes G_2}(u_i, v_j) = n_2 d_{G_1}(u_i) + n_1 d_{G_2}(v_j) - d_{G_1}(u_i) d_{G_2}(v_j)$. By the definition of the first multiplicative Zagreb index, we have

$$\begin{aligned}
&\prod_1 (G_1 \otimes G_2) \\
&= \prod_{(u_i, v_j) \in V(G_1 \otimes G_2)} d_{G_1 \otimes G_2}(u_i, v_j)^2 \\
&= \prod_{u_i \in V(G_1)} \prod_{v_j \in V(G_2)} \big(n_2 d_{G_1}(u_i) + n_1 d_{G_2}(v_j) - d_{G_1}(u_i) d_{G_2}(v_j)\big)^2 \\
&\le \left[\frac{\sum_{u_i \in V(G_1)} \sum_{v_j \in V(G_2)} (n_2 d_{G_1}(u_i) + n_1 d_{G_2}(v_j) - d_{G_1}(u_i) d_{G_2}(v_j))^2}{n_1 n_2} \right]^{n_1 n_2} \qquad (21) \\
&= \frac{1}{(n_1 n_2)^{n_1 n_2}} \big[n_2^3 M_1(G_1) + n_1^3 M_1(G_2) + M_1(G_1) M_1(G_2) \\
&\quad + 8 n_1 n_2 m_1 m_2 - 4 n_1 m_1 M_1(G_2) - 4 n_2 m_2 M_1(G_1) \big]^{n_1 n_2}.
\end{aligned}$$

The equality holds in (21) if and only if $d_{G_1}(u_i) = d_{G_1}(u_k)$, $u_i, u_k \in V(G_1)$ and $d_{G_2}(v_j) = d_{G_2}(v_\ell)$, $v_j, v_\ell \in V(G_2)$ (by Lemma 1), that is, both G_1 and G_2 are regular graphs, that is, $G_1 \circ G_2$ is a regular graph.

By the definition of the second multiplicative Zagreb index, we have

$$\begin{aligned}
\prod_2 (G_1 \otimes G_2) &= \prod_{(u_i, v_j)(u_k, v_\ell) \in E(G_1 \otimes G_2)} d_{G_1 \otimes G_2}(u_i, v_j) d_{G_1 \otimes G_2}(u_k, v_\ell) \\
&= \prod_{u_i \in V(G_1)} \prod_{v_j \in V(G_2)} P^P,
\end{aligned}$$

where

$$P = n_2 d_{G_1}(u_i) + n_1 d_{G_2}(v_j) - d_{G_1}(u_i) d_{G_2}(v_j).$$

Using the weighted arithmetic-geometric mean inequality in (2), $\prod_2 (G_1 \otimes G_2)$ is less than or equal to

$$\begin{aligned}
&\le \left[\frac{\sum_{u_i \in V(G_1)} \sum_{v_j \in V(G_2)} (n_2 d_{G_1}(u_i) + n_1 d_{G_2}(v_j) - d_{G_1}(u_i) d_{G_2}(v_j))^2}{\sum_{u_i \in V(G_1)} \sum_{v_j \in V(G_2)} P} \right]^{\sum_{u_i \in V(G_1)} \sum_{v_j \in V(G_2)} P} \qquad (22) \\
&= \left[\frac{M_1(G_1)(n_2^3 + M_1(G_2) - 4 n_2 m_2) + M_1(G_2)(n_1^2 - 4 n_1 m_1) + 8 n_1 n_2 m_1 m_2}{Q} \right]^Q,
\end{aligned}$$

where

$$Q = \sum_{u_i \in V(G_1)} \sum_{v_j \in V(G_2)} P = 2\big(n_2^2 m_1 + n_1^2 m_2 - 2 m_1 m_2\big).$$

Hence the first part of the proof is over.

The equality holds in (22) if and only if $d_{G_1}(u_i) = d_{G_1}(u_k)$, where $u_i, u_k \in V(G_1)$ and $d_{G_2}(v_j) = d_{G_2}(v_\ell)$, where $v_j, v_\ell \in V(G_2)$ (by Lemma 1), that is, both G_1 and G_2 are regular graphs, and so the graph $G_1 \circ G_2$ is regular. □

Example 5 $\prod_1(K_p \otimes C_q) = (pq - q + 2)^{2pq}$ and $\prod_2(K_p \otimes C_q) = (pq - q + 2)^{pq(pq-q+2)}$.

The *symmetric difference* $G_1 \oplus G_2$ of two graphs G_1 and G_2 is the graph with a vertex set $V(G_1) \times V(G_2)$ in which (u_i, v_j) is adjacent to (u_k, v_ℓ) whenever u_i is adjacent to u_k in G_1 or v_i is adjacent to v_ℓ in G_2, but not both. The degree of a vertex (u_i, v_j) of $G_1 \oplus G_2$ is given by

$$d_{G_1 \oplus G_2}(u_i, v_j) = n_2 d_{G_1}(u_i) + n_1 d_{G_2}(v_j) - 2d_{G_1}(u_i)d_{G_2}(v_j),$$

while the number of edges in $G_1 \oplus G_2$ is $n_1^2 m_2 + n_2^2 m_1 - 4m_1 m_2$.

Theorem 6 *The first and second multiplicative Zagreb indices of the symmetric difference $G_1 \oplus G_2$ of two graphs G_1 and G_2 are bounded above as follows*:

(i)

$$\prod_1(G_1 \oplus G_2) \le \frac{1}{(n_1 n_2)^{n_1 n_2}} \left[n_2^3 M_1(G_1) + n_1^3 M_1(G_2) + 4M_1(G_1)M_1(G_2) + 8n_1 n_2 m_1 m_2 - 8n_1 m_1 M_1(G_2) - 8n_2 m_2 M_1(G_1) \right]^{n_1 n_2}, \quad (23)$$

(ii)

$$\prod_2(G_1 \oplus G_2) \le \left[\frac{M_1(G_1)(n_2^3 + 4M_1(G_2) - 8n_2 m_2) + M_1(G_2)(n_1^3 - 8n_1 m_1) + 8n_1 n_2 m_1 m_2}{Q} \right]^Q, \quad (24)$$

where $Q = \sum_{u_i \in V(G_1)} \sum_{v_j \in V(G_2)} P = 2(n_2^2 m_1 + n_1^2 m_2 - 4m_1 m_2)$ and $M_1(G_i)$ is the first Zagreb index of G_i, for $i = 1, 2$. Moreover, the equalities in (23) *and* (24) *hold if and only if $G_1 \circ G_2$ is a regular graph.*

Proof We have

$$d_{G_1 \oplus G_2}(u_i, v_j) = n_2 d_{G_1}(u_i) + n_1 d_{G_2}(v_j) - 2d_{G_1}(u_i)d_{G_2}(v_j).$$

By the definition of the first multiplicative Zagreb index, we have

$$\begin{aligned} &\prod_1(G_1 \oplus G_2) \\ &= \prod_{(u_i, v_j) \in V(G_1 \oplus G_2)} d_{G_1 \otimes G_2}(u_i, v_j)^2 \\ &= \prod_{u_i \in V(G_1)} \prod_{v_j \in V(G_2)} \left(n_2 d_{G_1}(u_i) + n_1 d_{G_2}(v_j) - 2d_{G_1}(u_i)d_{G_2}(v_j) \right)^2 \end{aligned}$$

$$\begin{aligned}
&\leq \left[\frac{\sum_{u_i\in V(G_1)}\sum_{v_j\in V(G_2)}(n_2 d_{G_1}(u_i)+n_1 d_{G_2}(v_j)-2d_{G_1}(u_i)d_{G_2}(v_j))^2}{n_1 n_2}\right]^{n_1 n_2} \\
&= \frac{1}{(n_1 n_2)^{n_1 n_2}}\big[n_2^3 M_1(G_1)+n_1^3 M_1(G_2)+4M_1(G_1)M_1(G_2)+8n_1 n_2 m_1 m_2 \\
&\quad -8n_1 m_1 M_1(G_2)-8n_2 m_2 M_1(G_1)\big]^{n_1 n_2}.
\end{aligned} \tag{25}$$

The equality holds in (25) if and only if $d_{G_1}(u_i)=d_{G_1}(u_k)$, $u_i,u_k\in V(G_1)$ and $d_{G_2}(v_j)=d_{G_2}(v_\ell)$, $v_j,v_\ell\in V(G_2)$ (by Lemma 1), that is, both G_1 and G_2 are regular graphs, which implies that $G_1\circ G_2$ is a regular graph.

By the definition of the second multiplicative Zagreb index, we have

$$\begin{aligned}
\prod_2(G_1\oplus G_2) &= \prod_{(u_i,v_j)(u_k,v_\ell)\in E(G_1\otimes G_2)} d_{G_1\otimes G_2}(u_i,v_j)d_{G_1\otimes G_2}(u_k,v_\ell) \\
&= \prod_{u_i\in V(G_1)}\prod_{v_j\in V(G_2)} P^P,
\end{aligned}$$

where $P=n_2 d_{G_1}(u_i)+n_1 d_{G_2}(v_j)-2d_{G_1}(u_i)d_{G_2}(v_j)$.

Using the weighted arithmetic-geometric mean inequality in (2), we get

$$\begin{aligned}
&\prod_{u_i\in V(G_1)}\prod_{v_j\in V(G_2)} P^P \\
&\leq \left[\frac{\sum_{u_i\in V(G_1)}\sum_{v_j\in V(G_2)}(n_2 d_{G_1}(u_i)+n_1 d_{G_2}(v_j)-2d_{G_1}(u_i)d_{G_2}(v_j))^2}{\sum_{u_i\in V(G_1)}\sum_{v_j\in V(G_2)}P}\right]^{\sum_{u_i\in V(G_1)}\sum_{v_j\in V(G_2)}P} \\
&= \left[\frac{M_1(G_1)(n_2^3+4M_1(G_2)-8n_2 m_2)+M_1(G_2)(n_1^3-8n_1 m_1)+8n_1 n_2 m_1 m_2}{Q}\right]^Q,
\end{aligned} \tag{26}$$

where $Q=\sum_{u_i\in V(G_1)}\sum_{v_j\in V(G_2)}P=2(n_2^2 m_1+n_1^2 m_2-4m_1 m_2)$. First part of the proof is over.

The equality holds in (26) if and only if $d_{G_1}(u_i)=d_{G_1}(u_k)$, $u_i,u_k\in V(G_1)$ and $d_{G_2}(v_j)=d_{G_2}(v_\ell)$, $v_j,v_\ell\in V(G_2)$ (by Lemma 1), that is, both G_1 and G_2 are regular graphs, which implies that $G_1\circ G_2$ is a regular graph. □

Example 6 $\prod_1(G_1\oplus G_2)=(p+q-2)^{2pq}$ and $\prod_2(G_1\oplus G_2)=(p+q-2)^{pq(p+q-2)}$.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors completed the paper together. All authors read and approved the final manuscript.

Author details
[1]Department of Mathematics, Sungkyunkwan University, Suwon, 440-746, Republic of Korea. [2]Department of Mathematics, Faculty of Arts and Science, Uludağ University, Gorukle Campus, Bursa, 16059, Turkey. [3]Department of Mathematics, Faculty of Science, Selçuk University, Campus, Konya, 42075, Turkey.

Acknowledgements
Dedicated to Professor Hari M Srivastava.
All authors except the first one are partially supported by Research Project Offices of Uludağ (2012-15 and 2012-19) and Selçuk Universities. K.C. Das thanks for support the Sungkyunkwan University BK21 Project, BK21 Math Modeling HRD Div. Sungkyunkwan University, Suwon, Republic of Korea.

Received: 19 December 2012 Accepted: 15 February 2013 Published: 5 March 2013

References

1. Todeschini, R, Ballabio, D, Consonni, V: Novel molecular descriptors based on functions of new vertex degrees. In: Gutman, I, Furtula, B (eds.) Novel Molecular Structure Descriptors - Theory and Applications I, pp. 73-100. Univ. Kragujevac, Kragujevac (2010)
2. Todeschini, R, Consonni, V: New local vertex invariants and molecular descriptors based on functions of the vertex degrees. MATCH Commun. Math. Comput. Chem. **64**, 359-372 (2010)
3. Eliasi, M, Iranmanesh, A, Gutman, I: Multiplicative versions of first Zagreb index. MATCH Commun. Math. Comput. Chem. **68**, 217-230 (2012)
4. Gutman, I: Multiplicative Zagreb indices of trees. Bull. Soc. Math. Banja Luka **18**, 17-23 (2011)
5. Liu, J, Zhang, Q: Sharp upper bounds on multiplicative Zagreb indices. MATCH Commun. Math. Comput. Chem. **68**, 231-240 (2012)
6. Xu, K, Hua, H: A unified approach to extremal multiplicative Zagreb indices for trees, unicyclic and bicyclic graphs. MATCH Commun. Math. Comput. Chem. **68**, 241-256 (2012)
7. Xu, K, Das, KC: Trees, unicyclic and bicyclic graphs extremal with respect to multiplicative sum Zagreb index. MATCH Commun. Math. Comput. Chem. **68**(1), 257-272 (2012)
8. Bondy, JA, Murty, USR: Graph Theory with Applications. Macmillan Co., New York (1976)
9. Khalifeh, MH, Azari, HY, Ashrafi, AR: The hyper-Wiener index of graph operations. Comput. Math. Appl. **56**, 1402-1407 (2008)
10. Khalifeh, MH, Azari, HY, Ashrafi, AR: The first and second Zagreb indices on of some graph operations. Discrete Appl. Math. **157**, 804-811 (2009)
11. Imrich, W, Klavžar, S: Product Graphs: Structure and Recognition. Wiley, New York (2000)
12. Harary, F: Graph Theory, p. 22. Addison-Wesley, Reading (1994)

doi:10.1186/1029-242X-2013-90
Cite this article as: Das et al.: **The multiplicative Zagreb indices of graph operations.** *Journal of Inequalities and Applications* 2013 **2013**:90.

Submit your manuscript to a SpringerOpen journal and benefit from:

► Convenient online submission
► Rigorous peer review
► Immediate publication on acceptance
► Open access: articles freely available online
► High visibility within the field
► Retaining the copyright to your article

Submit your next manuscript at ► springeropen.com